

Pari-GP reference card

(PARI-GP version 2.15.3)

Note: optional arguments are surrounded by braces {}.

To start the calculator, type its name in the terminal: **gp**

To exit **gp**, type **quit**, **\q**, or **<C-D>** at prompt.

Help

describe function	?function
extended description	??keyword
list of relevant help topics	???pattern
name of GP-1.39 function f in GP-2.*	whatnow(f)

Input/Output

previous result, the result before	%, %`, %`` , etc.
n -th result since startup	% n
separate multiple statements on line	;
extend statement on additional lines	\
extend statements on several lines	{ seq_1 ; seq_2 ;}
comment	/* ... */
one-line comment, rest of line ignored	\\ ...

Metacommands & Defaults

set default d to val	default({ d },{ val })
toggle timer on/off	#
print time for last result	##
print defaults	\d
set debug level to n	\g n
set memory debug level to n	\gm n
set n significant digits / bits	\p n , \pb n
set n terms in series	\ps n
quit GP	\q
print the list of PARI types	\t
print the list of user-defined functions	\u
read file into GP	\r <i>filename</i>
set debuglevel for domain D to n	setdebug(D,n)

Debugger / break loop

get out of break loop	break or <C-D>
go up/down n frames	dbg_up({ n }), dbg_down
set break point	breakpoint()
examine object o	dbg_x(o)
current error data	dbg_err()
number of objects on heap and their size	getheap()
total size of objects on PARI stack	getstack()

PARI Types & Input Formats

t_INT . Integers; hex, binary	± 31 ; $\pm 0x1F$, $\pm 0b101$
t_REAL . Reals	± 3.14 , 6.022 E23
t_INTMOD . Integers modulo m	Mod(n,m)
t_FRAC . Rational Numbers	n/m
t_FFELT . Elt in finite field \mathbf{F}_q	ffgen(q , 't)
t_COMPLEX . Complex Numbers	$x + y * I$
t_PADIC . p -adic Numbers	$x + O(p^k)$
t_QUAD . Quadratic Numbers	$x + y * \text{quadgen}(D, \{ 'w \})$
t_POLMOD . Polynomials modulo g	Mod(f,g)
t_POL . Polynomials	$a * x^n + \dots + b$
t_SER . Power Series	$f + O(x^k)$
t_RFRAC . Rational Functions	f/g
t_QFB . Binary quadratic form	Qfb(a,b,c)
t_VEC/t_COL . Row/Column Vectors	[x,y,z], [x,y,z]~
t_VEC integer range	[1..10]

t_VECSMALL . Vector of small ints	Vecsmall([x,y,z])
t_MAT . Matrices	[$a,b;c,d$]
t_LIST . Lists	List([x,y,z])
t_STR . Strings	"abc"
t_INFINITY . $\pm\infty$	+oo, -oo

Reserved Variable Names

$\pi \approx 3.14$, $\gamma \approx 0.57$, $C \approx 0.91$, $I = \sqrt{-1}$	Pi, Euler, Catalan, I
Landau's big-oh notation	O

Information about an Object, Precision

PARI type of object x	type(x)
length of x / size of x in memory	# x , sizebyte(x)
real precision / bit precision of x	precision(x), bitprecision(x)
p -adic, series prec. of x	padicprec(x,p), serprec(x,v)
current dynamic precision	getlocalprec, getlocalbitprec

Operators

basic operations	+, -, *, /, ^, sqr
$i \leftarrow i+1$, $i \leftarrow i-1$, $i \leftarrow i*j$, ...	i++, i--, i*=j,...
Euclidean quotient, remainder	$x \backslash y$, $x \backslash y$, $x \% y$, divrem(x,y)
shift x left or right n bits	$x << n$, $x >> n$ or shift($x, \pm n$)
multiply by 2^n	shiftmul(x,n)
comparison operators	<=, <, >=, >, ==, !=, ==, lex, cmp
boolean operators (or, and, not)	, &&, !
bit operations	bitand, bitneg, bitor, bitxor, bitnegimply
maximum/minimum of x and y	max(x,y), min(x,y)
sign of x (gives $-1, 0, 1$)	sign(x)
binary exponent of x	exponent(x)
derivative of f , 2nd derivative, etc.	f' , f'' , ...
differential operator	diffop($f,v,d,\{n=1\}$)
quote operator (formal variable)	'x
assignment	x = value
simultaneous assignment $x \leftarrow v[1]$, $y \leftarrow v[2]$	[x,y] = v

Select Components

Caveat: components start at index $n = 1$.

n -th component of x	component(x,n)
n -th component of vector/list x	$x[n]$
components $a, a+1, \dots, b$ of vector x	$x[a..b]$
(m,n) -th component of matrix x	$x[m,n]$
row m or column n of matrix x	$x[m,]$, $x[,n]$
numerator/denominator of x	numerator(x), denominator(x)

Random Numbers

random integer/prime in $[0,N[$	random(N), randomprime(N)
get/set random seed	getrand, setrand(s)

Conversions

to vector, matrix, vec. of small ints	Col/Vec, Mat, Vecsmall
to list, set, map, string	List, Set, Map, Str
create $(x \bmod y)$	Mod(x,y)
make x a polynomial of v	Pol($x,\{v\}$)
variants of Pol <i>et al.</i> , in reverse order	Polrev, Vecrev, Colrev
make x a power series of v	Ser($x,\{v\}$)
convert x to simplest possible type	simplify(x)
object x with real precision n	precision(x,n)
object x with bit precision n	bitprecision(x,n)
set precision to p digits in dynamic scope	localprec(p)
set precision to p bits in dynamic scope	localbitprec(p)

Character strings

convert to TeX representation	strtex(x)
string from bytes / from format+args	strchr, sprintf
split string / join strings	strsplit, strjoin
convert time t ms. to h, m, s, ms format	strtime(t)
Conjugates and Lifts	
conjugate of a number x	conj(x)
norm of x , product with conjugate	norm(x)
L^p norm of x (L^∞ if no p)	normlp($x,\{p\}$)
square of L^2 norm of x	norml2(x)
lift of x from Mods and p -adics	lift, centerlift(x)
recursive lift	liftall
lift all t_INT and t_PADIC (\rightarrow t_INT)	liftint
lift all t_POLMOD (\rightarrow t_POL)	liftpol

Lists, Sets & Maps

Sets (= row vector with strictly increasing entries w.r.t. **cmp**)

intersection of sets x and y	setintersect(x,y)
set of elements in x not belonging to y	setminus(x,y)
symmetric difference $x \Delta y$	setdelta(x,y)
union of sets x and y	setunion(x,y)
does y belong to the set x	setsearch($x,y,\{flag\}$)
set of all $f(x,y)$, $x \in X$, $y \in Y$	setbinop(f,X,Y)
is x a set ?	setisset(x)

Lists. create empty list: $L = \text{List}()$

append x to list L	listput($L,x,\{i\}$)
remove i -th component from list L	listpop($L,\{i\}$)
insert x in list L at position i	listinsert(L,x,i)
sort the list L in place	listsort($L,\{flag\}$)

Maps. create empty dictionary: $M = \text{Map}()$

attach value v to key k	mapput(M,k,v)
recover value attach to key k or error	mapget(M,k)
is key k in the dict? (set v to $M(k)$)	mapisdefined($M,k,\{\&v\}$)
remove k from map domain	mapdelete(M,k)

GP Programming

User functions and closures

x,y are formal parameters; y defaults to Pi if parameter omitted;
 z,t are local variables (lexical scope), z initialized to 1.

fun(x, y=Pi) = my(z=1, t); *seq*
fun = (x, y=Pi) -> my(z=1, t); *seq*

attach help message h to s	addhelp(s,h)
undefine symbol s (also kills help)	kill(s)
Control Statements (X : formal parameter in expression seq)	
if $a \neq 0$, evaluate seq_1 , else seq_2	if($a,\{seq_1\},\{seq_2\}$)
eval. seq for $a \leq X \leq b$	for($X = a,b,seq$)
...for $X \in v$	foreach(v,X,seq)
...for primes $a \leq X \leq b$	forprime($X = a,b,seq$)
...for primes $\equiv a \pmod q$	forprimestep($X = a,b,q,seq$)
...for composites $a \leq X \leq b$	forcomposite($X = a,b,seq$)
...for $a \leq X \leq b$ stepping s	forstep($X = a,b,s,seq$)
...for X dividing n	fordiv(n,X,seq)
... $X = [n, factor(n)]$, $a \leq n \leq b$	forfactored($X = a,b,seq$)
...as above, n squarefree	forsquarefree($X = a,b,seq$)
... $X = [d, factor(d)]$, $d \mid n$	fordivfactored(n,X,seq)
multivariable for , lex ordering	forvec($X = v,seq$)

```

loop over partitions of  $n$ 
... permutations of  $S$ 
... subsets of  $\{1, \dots, n\}$ 
...  $k$ -subsets of  $\{1, \dots, n\}$ 
... vectors  $v, q(v) \leq B$ ;  $q > 0$ 
...  $H < G$  finite abelian group
evaluate  $seq$  until  $a \neq 0$ 
while  $a \neq 0$ , evaluate  $seq$ 
exit  $n$  innermost enclosing loops
start new iteration of  $n$ -th enclosing loop
return  $x$  from current subroutine
Exceptions, warnings
raise an exception / warning
type of error message  $E$ 
try  $seq_1$ , evaluate  $seq_2$  on error
Functions with closure arguments / results
number of arguments of  $f$ 
select from  $v$  according to  $f$ 
apply  $f$  to all entries in  $v$ 
evaluate  $f(a_1, \dots, a_n)$ 
evaluate  $f(\dots f(f(a_1, a_2), a_3) \dots, a_n)$ 
calling function as closure
Sums & Products
sum  $X = a$  to  $X = b$ , initialized at  $x$ 
sum entries of vector  $v$ 
product of all vector entries
sum  $expr$  over divisors of  $n$ 
... assuming  $expr$  multiplicative
product  $a \leq X \leq b$ , initialized at  $x$ 
product over primes  $a \leq X \leq b$ 
Sorting
sort  $x$  by  $k$ -th component
min.  $m$  of  $x$  ( $m = x[i]$ ), max.
does  $y$  belong to  $x$ , sorted wrt.  $f$ 
 $\prod g^x \rightarrow$  factorization ( $\Rightarrow$  sorted, unique  $g$ )
Input/Output
print with/without  $\backslash n$ , TeX format
pretty print matrix
print fields with separator
formatted printing
write  $args$  to file
write  $x$  in binary format
read file into GP
... return as vector of lines
... return as vector of strings
read a string from keyboard
Files and file descriptors
File descriptors allow efficient small consecutive reads or writes
from or to a given file. The argument  $n$  below is always a descriptor,
attached to a file in r(ead), w(rite) or a(ppend) mode.
get descriptor  $n$  for file  $path$  in given  $mode$ 
... from shell  $cmd$  output (pipe)
close descriptor
commit pending write operations
read logical line from file
... raw line from file
write  $s \backslash n$  to file
... write  $s$  to file

```

```

forpart( $p = n, seq$ )
forperm( $S, p, seq$ )
forsubset( $n, p, seq$ )
forsubset( $[n, k], p, seq$ )
forqfvec( $v, q, b, seq$ )
forsubgroup( $H = G$ )
until( $a, seq$ )
while( $a, seq$ )
break( $\{n\}$ )
next( $\{n\}$ )
return( $\{x\}$ )

error(), warning()
errname( $E$ )
iferr( $seq_1, E, seq_2$ )

Results
arity( $f$ )
select( $f, v$ )
apply( $f, v$ )
call( $f, a$ )
fold( $f, a$ )
self()

sum( $X = a, b, expr, \{x\}$ )
vecsum( $v$ )
vecprod( $v$ )
sumdiv( $n, X, expr$ )
sumdivmult( $n, X, expr$ )
prod( $X = a, b, expr, \{x\}$ )
prodeuler( $X = a, b, expr$ )

vecsrt( $x, \{k\}, \{fl = 0\}$ )
vecmin( $x, \{&i\}$ ), vecmax
vecsearch( $x, y, \{f\}$ )
matreduce( $m$ )

print, print1, printtex
printp
printsep( $sep, \dots$ ), printsep1
printf()

write, write1, writetex( $file, args$ )
writebin( $file, x$ )
read( $\{file\}$ )
readvec( $\{file\}$ )
readstr( $\{file\}$ )
input()

fileclose( $n$ )
fileflush( $n$ )
fileread( $n$ )
filereadstr( $n$ )
filewrite( $n, s$ )
filewrite1( $n, s$ )

```

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Timers

CPU time in ms and reset timer
CPU time in ms since gp startup
time in ms since UNIX Epoch
timeout command after s seconds

Interface with system

allocates a new stack of s bytes
alias old to new
install function from library
execute system command a
... and feed result to GP
... returning GP string
get \$VAR from environment
expand env. variable in string

```

gettime()
getabstime()
getwalltime()
alarm( $s, expr$ )

allocatemem( $\{s\}$ )
alias( $new, old$ )
install( $f, code, \{gpf\}, \{lib\}$ )
system( $a$ )
extern( $a$ )
externstr( $a$ )
getenv("VAR")
strexpand( $x$ )

```

Parallel evaluation

These functions evaluate their arguments in parallel (pthreads or MPI); args. must not access global variables (use **export** for this) and must be free of side effects. Enabled if threading engine is not *single* in gp header.

```

evaluate  $f$  on  $x[1], \dots, x[n]$ 
evaluate closures  $f[1], \dots, f[n]$ 
as select
as sum
as vector
eval  $f$  for  $i = a, \dots, b$ 
... for each element  $x$  in  $v$ 
... for  $p$  prime in  $[a, b]$ 
... for  $p = a \bmod q$ 
... multivariate
parforvec( $X = v, f, \{r\}, \{f_2\}, \{flag\}$ )
export  $x$  to parallel world
... all dynamic variables
frees exported value  $x$ 
... all exported values

```

```

parapply( $f, x$ )
pareval( $f$ )
parselect( $f, A, \{flag\}$ )
parsum( $i = a, b, expr$ )
parvector( $n, i, \{expr\}$ )
parfor( $i = a, \{b\}, f, \{r\}, \{f_2\}$ )
parforeach( $v, x, f, \{r\}, \{f_2\}$ )
parforprime( $p = a, \{b\}, f, \{r\}, \{f_2\}$ )
parforprimestep( $p = a, \{b\}, q, f, \{r\}, \{f_2\}$ )

```

Linear Algebra

dimensions of matrix x
multiply two matrices
... assuming result is diagonal
concatenation of x and y
extract components of x
transpose of vector or matrix x
adjoint of the matrix x
eigenvectors/values of matrix x
characteristic/minimal polynomial of x
trace/determinant of matrix x
permanent of matrix x
Frobenius form of x
QR decomposition
apply **matqr**'s transform to v

```

matsize( $x$ )
 $x * y$ 
matmultodiagonal( $x, y$ )
concat( $x, \{y\}$ )
vecextract( $x, y, \{z\}$ )
 $x \sim$ , mattranspose( $x$ )
matadjoint( $x$ )
mateigen( $x$ )
charpoly( $x$ ), minpoly( $x$ )
trace( $x$ ), matdet( $x$ )
matpermanent( $x$ )
matfrobenius( $x$ )
matqr( $x$ )
mathouseholder( $Q, v$ )

```

Constructors & Special Matrices

```

{ $g(x): x \in v$  s.t.  $f(x)$ }
{ $x: x \in v$  s.t.  $f(x)$ }
{ $g(x): x \in v$ }
row vec. of  $expr$  eval'ed at  $1 \leq i \leq n$ 
col. vec. of  $expr$  eval'ed at  $1 \leq i \leq n$ 
vector of small ints

```

```

[g(x) | x <- v, f(x)]
[x | x <- v, f(x)]
[g(x) | x <- v]
vector( $n, \{i\}, \{expr\}$ )
vectorv( $n, \{i\}, \{expr\}$ )
vectorsmall( $n, \{i\}, \{expr\}$ )

```

```

[ $c, c \cdot x, \dots, c \cdot x^n$ ]
[ $1, 2^x, \dots, n^x$ ]
matrix  $1 \leq i \leq m, 1 \leq j \leq n$ 
define matrix by blocks
diagonal matrix with diagonal  $x$ 
is  $x$  diagonal?
 $x \cdot \text{matdiagonal}(d)$ 
 $n \times n$  identity matrix
Hessenberg form of square matrix  $x$ 
 $n \times n$  Hilbert matrix  $H_{ij} = (i + j - 1)^{-1}$ 
 $n \times n$  Pascal triangle
companion matrix to polynomial  $x$ 
Sylvester matrix of  $x$  and  $y$ 

```

Gaussian elimination

```

kernel of matrix  $x$ 
intersection of column spaces of  $x$  and  $y$ 
solve  $MX = B$  ( $M$  invertible)
one sol of  $M * X = B$ 
basis for image of matrix  $x$ 
columns of  $x$  not in matimage
supplement columns of  $x$  to get basis
rows, cols to extract invertible matrix
rank of the matrix  $x$ 
solve  $MX = B \bmod D$ 
image mod  $D$ 
kernel mod  $D$ 
inverse mod  $D$ 
determinant mod  $D$ 

```

Lattices & Quadratic Forms

Quadratic forms

```

evaluate  ${}^t x Q y$ 
evaluate  ${}^t x Q x$ 
signature of quad form  ${}^t y * x * y$ 
decomp into squares of  ${}^t y * x * y$ 
eigenvalues/vectors for real symmetric  $x$ 

```

HNF and SNF

```

upper triangular Hermite Normal Form
HNF of  $x$  where  $d$  is a multiple of  $\det(x)$ 
multiple of  $\det(x)$ 
HNF of  $(x \mid \text{diagonal}(D))$ 
elementary divisors of  $x$ 
 $q$ -rank from elementary divisors
elementary divisors of  $\mathbf{Z}[a]/(f'(a))$ 
integer kernel of  $x$ 
 $\mathbf{Z}$ -module  $\leftrightarrow$   $\mathbf{Q}$ -vector space

```

Lattices

```

LLL-algorithm applied to columns of  $x$ 
... for Gram matrix of lattice
find up to  $m$  sols of qfnorm( $x, y) \leq b$ 
 $v, v[i] :=$  number of  $y$  s.t. qfnorm( $x, y) = i$ 
perfection rank of  $x$ 
find isomorphism between  $q$  and  $Q$ 
precompute for isomorphism test with  $q$ 
automorphism group of  $q$ 

```

Based on an earlier version by Joseph H. Silverman
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Send comments and corrections to (Karim.Belabas@math.u-bordeaux.fr)

convert **qfauto** for GAP/Magma **qfautoexport**($G, \{flag\}$)
orbits of V under $G \subset \text{GL}(V)$ **qforbits**(G, V)

Polynomials & Rational Functions

all defined polynomial variables **variables**()
get var. of highest priority (higher than v) **varhigher**($name, \{v\}$)
... of lowest priority (lower than v) **varlower**($name, \{v\}$)

Coefficients, variables and basic operators

degree of f **poldegree**(f)
coef. of degree n of f , leading coef. **polcoef**(f, n), **pollead**
main variable / all variables in f **variable**(f), **variables**(f)
replace x by y in f **subst**(f, x, y)
evaluate f replacing vars by their value **eval**(f)
replace polynomial expr. $T(x)$ by y in f **substpol**(f, T, y)
replace x_1, \dots, x_n by y_1, \dots, y_n in f **substvec**(f, x, y)
 $f \in A[x]$; reciprocal polynomial $x^{\deg f} f\left(\frac{1}{x}\right)$ **polrecip**(f)
gcd of coefficients of f **content**(f)
derivative of f w.r.t. x **deriv**($f, \{x\}$)
... n -th derivative of f **derivn**($f, n, \{x\}$)
formal integral of f w.r.t. x **intformal**($f, \{x\}$)
formal sum of f w.r.t. x **sumformal**($f, \{x\}$)

Constructors & Special Polynomials

interpolation polynomial at $(x[1], y[1]), \dots, (x[n], y[n])$, evaluated at t , with error estimate e **polinterpolate**($x, \{y\}, \{t\}, \{&e\}$)
 $T_n/U_n, H_n$ **polchebyshev**(n), **polhermite**(n)
 $P_n, L_n^{(\alpha)}$ **pollegendre**(n), **pollaguerre**(n, a)
 n -th cyclotomic polynomial Φ_n **polcyclo**(n)
return n if $f = \Phi_n$, else 0 **poliscyclo**(f)
is f a product of cyclotomic polynomials? **poliscycloprod**(f)
Zagier's polynomial of index (n, m) **polzagier**(n, m)

Resultant, elimination

discriminant of polynomial f **poldisc**(f)
find factors of **poldisc**(f) **poldiscfactors**(f)
resultant $R = \text{Res}_v(f, g)$ **polresultant**($f, g, \{v\}$)
 $[u, v, R], xu + yv = \text{Res}_v(f, g)$ **polresultanttext**($x, y, \{v\}$)
solve Thue equation $f(x, y) = a$ **thue**($t, a, \{sol\}$)
initialize t for Thue equation solver **thueinit**(f)

Roots and Factorization (Complex/Real)

complex roots of f **polroots**(f)
bound complex roots of f **polrootsbound**(f)
number of real roots of f (in $[a, b]$) **polsturm**($f, \{[a, b]\}$)
real roots of f (in $[a, b]$) **polrootsreal**($f, \{[a, b]\}$)
complex embeddings of **t_POLMOD** z **conjvec**(z)

Roots and Factorization (Finite fields)

factor f mod p , roots **factormod**(f, p), **polrootsmod**
factor f over $\mathbf{F}_p[x]/(T)$, roots **factormod**($f, [T, p]$), **polrootsmod**
squarefree factorization of f in $\mathbf{F}_q[x]$ **factormodSQF**($f, \{D\}$)
distinct degree factorization of f in $\mathbf{F}_q[x]$ **factormodDDF**($f, \{D\}$)
factor n -th cyclotomic pol. Φ_n mod p **factormodcyclo**(n, p)

Roots and Factorization (p -adic fields)

factor f over \mathbf{Q}_p , roots **factorpadic**(f, p, r), **polrootspadic**
 p -adic root of f congruent to a mod p **padicappr**(f, a)
Newton polygon of f for prime p **newtonpoly**(f, p)
Hensel lift $A/\text{lc}(A) = \prod_i B[i]$ mod p^e **polhensellift**(A, B, p, e)
 $T = \prod (x - z_i) \mapsto \prod [x - \omega(z_i)] \in \mathbf{Z}_p[x]$ **polteichmuller**(T, p, e)
extensions of \mathbf{Q}_p of degree N **padicfields**(p, N)

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Roots and Factorization (Miscellaneous)

symmetric powers of roots of f up to n **polsym**(f, n)
Graeffe transform of $f, g(x^2) = f(x)f(-x)$ **polgraeffe**(f)
factor f over coefficient field **factor**(f)
cyclotomic factors of $f \in \mathbf{Q}[X]$ **polcyclofactors**(f)

Finite Fields

A finite field is encoded by any element (**t_FFELT**).
find irreducible $T \in \mathbf{F}_p[x]$, $\deg T = n$ **ffinit**($p, n, \{x\}$)
Create t in $\mathbf{F}_q \simeq \mathbf{F}_p[t]/(T)$ **t = ffgn**($T, 't$)
... indirectly, with implicit T **t = ffgn**($q, 't$); **T = t.mod**
map m from $\mathbf{F}_q \ni a$ to $\mathbf{F}_{q^k} \ni b$ **m = ffgend**(a, b)
build $K = \mathbf{F}_q[x]/(P)$ extending $\mathbf{F}_q \ni a$, **ffextend**(a, P)
evaluate map m on x **ffmap**(m, x)
inverse map of m **ffinvmap**(m)
compose maps $m \circ n$ **ffcompomap**(m, n)
 x as polmod over codomain of map m **ffmaprel**(m, x)
 F^n over $\mathbf{F}_q \ni a$ **fffrobenius**(a, n)
 $\# \{ \text{monic irred. } T \in \mathbf{F}_q[x], \deg T = n \}$ **ffnbirred**(q, n)

Formal & p-adic Series

truncate power series or p -adic number **truncate**(x)
valuation of x at p **valuation**(x, p)
Dirichlet and Power Series
Taylor expansion around 0 of f w.r.t. x **taylor**(f, x)
Laurent series of closure F up to x^k **laurentseries**(f, k)
 $\sum a_k b_k t^k$ from $\sum a_k t^k$ and $\sum b_k t^k$ **serconvol**(a, b)
 $f = \sum a_k t^k$ from $\sum (a_k/k!) t^k$ **serlaplace**(f)
reverse power series F so $F(f(x)) = x$ **serreverse**(f)
remove terms of degree $< n$ in f **serchop**(f, n)
Dirichlet series multiplication / division **dirmul, dirdiv**(x, y)
Dirichlet Euler product (b terms) **direuler**($p = a, b, expr$)

Transcendental and p -adic Functions

real, imaginary part of x **real**(x), **imag**(x)
absolute value, argument of x **abs**(x), **arg**(x)
square/nth root of x **sqrt**(x), **sqrtn**($x, n, \{&z\}$)
all n -th roots of 1 **rootsof1**(n)
FFT of $[f_0, \dots, f_{n-1}]$ **w = fftinit**(n), **fft/fftin**(w, f)
trig functions **sin, cos, tan, cotan, sinc**
inverse trig functions **asin, acos, atan**
hyperbolic functions **sinh, cosh, tanh, cotanh**
inverse hyperbolic functions **asinh, acosh, atanh**
 $\log(x), \log(1+x), e^x, e^x - 1$ **log, loglp, exp, expm1**
Euler Γ function, $\log \Gamma, \Gamma'/\Gamma$ **gamma, lngamma, psi**
half-integer gamma function $\Gamma(n + 1/2)$ **gammah**(n)
Riemann's zeta $\zeta(s) = \sum n^{-s}$ **zeta**(s)
 $\sum_{1 \leq n \leq N} n^s$ **dirpowerssum**(N, s)
Hurwitz's $\zeta(s, x) = \sum (n+x)^{-s}$ **zetahurwitz**(s, x)
Lerch $\Phi(z, s, x) = \sum z^n (n+x)^{-s}$ **lerchphi**(z, s, x)
Lerch $L(s, x, t) = \Phi(e^{2i\pi t}, s, x)$ **lerchzeta**(s, x, t)
multiple zeta value (MZV), $\zeta(s_1, \dots, s_k)$ **zetamult**($s, \{T\}$)
all MZVs for weight $\sum s_i = n$ **zetamultall**(n)
convert MZV id to $[s_1, \dots, s_k]$ **zetamultconvert**($f, \{flag\}$)
MZV dual sequence **zetamultdual**(s)
multiple polylog $Li_{s_1, \dots, s_k}(z_1, \dots, z_k)$ **polylogmult**(s, z)

incomplete Γ function ($y = \Gamma(s)$) **incgam**($s, x, \{y\}$)
complementary incomplete Γ **incgamc**(s, x)
 $\int_x^\infty e^{-t} dt/t, (2/\sqrt{\pi}) \int_x^\infty e^{-t^2} dt$ **eint1, erfc**
elliptic integral of 1st and 2nd kind **ellK**(k), **ellE**(k)
dilogarithm of x **dilog**(x)
 m -th polylogarithm of x **polylog**($m, x, \{flag\}$)
 U -confluent hypergeometric function **hyperu**(a, b, u)
Hypergeometric ${}_pF_q(A, B; z)$ **hypergeom**(A, B, z)
Bessel $J_n(x), J_{n+1/2}(x)$ **besselj**(n, x), **besseljh**(n, x)
Bessel $I_\nu, K_\nu, H_\nu^1, H_\nu^2, Y_\nu$ **(bessel)i, k, h1, h2, y**
 k -th zero of $J_\nu(x)$ **besseljzero**($nu, \{k = 1\}$)
 k -th zero of $Y_\nu(x)$ **besselyzero**($nu, \{k = 1\}$)
Airy functions $A_i(x), B_i(x)$ **airy**(x)
Lambert $W: x$ s.t. $xe^x = y$ **lambertw**(y)
Teichmuller character of p -adic x **teichmuller**(x)

Iterations, Sums & Products

Numerical integration for meromorphic functions

Behaviour at endpoint for Double Exponential (DE) methods: either a scalar ($a \in \mathbf{C}$, regular) or $\pm\infty$ (decreasing at least as x^{-2}) or
 $(x-a)^{-\alpha}$ singularity **[a, a]**
exponential decrease $e^{-\alpha|x|}$ **[$\pm\infty, a$], $\alpha > 0$**
slow decrease $|x|^\alpha$ **$\dots \alpha < -1$**
oscillating as $\cos(kx)$ **$\alpha = k\mathbf{I}, k > 0$**
oscillating as $\sin(kx)$ **$\alpha = -k\mathbf{I}, k > 0$**

numerical integration **intnum**($x = a, b, f, \{T\}$)
weights T for **intnum** **intnuminit**($a, b, \{m\}$)
weights T incl. kernel K **intfuncinit**($t = a, b, K, \{m\}$)
integrate $(2i\pi)^{-1} f$ on circle $|z-a| = R$ **intcirc**($x = a, R, f, \{T\}$)
Other integration methods
 n -point Gauss-Legendre **intnumgauss**($x = a, b, f, \{n\}$)
weights for n -point Gauss-Legendre **intnumgaussinit**($\{n\}$)
quasi-periodic function, period $2H$ **intnumosc**($x = a, f, H$)
Romberg (low accuracy) **intnumromb**($x = a, b, f, \{flag\}$)

Numerical summation

sum of series $f(n), n \geq a$ (low accuracy) **suminf**($n = a, expr$)
sum of alternating/positive series **sumalt, sumpos**
sum of series using Euler-Maclaurin **sumnum**($n = a, f, \{T\}$)
... Sidi summation **sumnumsidi**($n = a, f$)
 $\sum_{n \geq a} F(n)$, F rational function **sumnumrat**(F, a)
 $\dots \sum_{p \geq a} F(p^s)$ **sumeulerrat**($F, \{s = 1\}, \{a = 2\}$)
weights for **sumnum**, a as in DE **sumnuminit**($\{\infty, a\}$)
sum of series by Monien summation **sumnummonien**($n = a, f, \{T\}$)
weights for **sumnummonien** **sumnummonieninit**($\{\infty, a\}$)
sum of series using Abel-Plana **sumnumap**($n = a, f, \{T\}$)
weights for **sumnumap**, a as in DE **sumnumapinit**($\{\infty, a\}$)
sum of series using Lagrange **sumnumlagrange**($n = a, f, \{T\}$)
weights for **sumnumlagrange** **sumnumlagrangeinit**

Products

product $a \leq X \leq b$, initialized at x **prod**($X = a, b, expr, \{x\}$)
product over primes $a \leq X \leq b$ **prodeuler**($X = a, b, expr$)
infinite product $a \leq X \leq \infty$ **prodinf**($X = a, expr$)
 $\prod_{n \geq a} F(n)$, F rational function **prodnumrat**(F, a)
 $\prod_{p \geq a} F(p^s)$ **prodeulerrat**($F, \{s = 1\}, \{a = 2\}$)

Other numerical methods

real root of f in $[a, b]$; bracketed root	<code>solve($X = a, b, f$)</code>
...interval splitting, step s	<code>solvestep($X = a, b, s, f, \{flag = 0\}$)</code>
limit of $f(t)$, $t \rightarrow \infty$	<code>limitnum($f, \{\alpha\}$)</code>
asymptotic expansion of f (rational)	<code>asypnum($f, \{\alpha\}$)</code>
... $N + 1$ terms as floats	<code>asypnumraw($f, N, \{\alpha\}$)</code>
numerical derivation w.r.t x : $f'(a)$	<code>derivnum($x = a, f$)</code>
evaluate continued fraction F at t	<code>contfraceval($F, t, \{L\}$)</code>
power series to cont. fraction (L terms)	<code>contfracinit($S, \{L\}$)</code>
Padé approximant (deg. denom. $\leq B$)	<code>bestapprPade($S, \{B\}$)</code>

Elementary Arithmetic Functions

vector of binary digits of $ x $	<code>binary(x)</code>
bit number n of integer x	<code>bittest(x, n)</code>
Hamming weight of integer x	<code>hammingweight(x)</code>
digits of integer x in base B	<code>digits($x, \{B = 10\}$)</code>
sum of digits of integer x in base B	<code>sumdigits($x, \{B = 10\}$)</code>
integer from digits	<code>fromdigits($v, \{B = 10\}$)</code>
ceiling/floor/fractional part	<code>ceil, floor, frac</code>
round x to nearest integer	<code>round($x, \{\&e\}$)</code>
truncate x	<code>truncate($x, \{\&e\}$)</code>
gcd/LCM of x and y	<code>gcd(x, y), lcm(x, y)</code>
gcd of entries of a vector/matrix	<code>content(x)</code>

Primes and Factorization

extra prime table	<code>addprimes()</code>
add primes in v to prime table	<code>addprimes(v)</code>
remove primes from prime table	<code>removeprimes(v)</code>
Chebyshev $\pi(x)$, n -th prime p_n	<code>primepi(x), prime(n)</code>
vector of first n primes	<code>primes(n)</code>
smallest prime $\geq x$	<code>nextprime(x)</code>
largest prime $\leq x$	<code>precprime(x)</code>
factorization of x	<code>factor($x, \{lim\}$)</code>
...selecting specific algorithms	<code>factorint($x, \{flag = 0\}$)</code>
$n = df^2$, d squarefree/fundamental	<code>core($n, \{fl\}$), coredisc</code>
certificate for (prime) N	<code>primecert(N)</code>
verifies a certificate c	<code>primecertisvalid(c)</code>
convert certificate to Magma/PRIMO	<code>primecertexport</code>
recover x from its factorization	<code>factorback($f, \{e\}$)</code>
$x \in \mathbf{Z}$, $ x \leq X$, $\gcd(N, P(x)) \geq N$	<code>zncoppersmith($P, N, X, \{B\}$)</code>
divisors of N in residue class r mod s	<code>divisorslensstra(N, r, s)</code>

Divisors and multiplicative functions

number of prime divisors $\omega(n)$ / $\Omega(n)$	<code>omega(n), bigomega</code>
divisors of n / number of divisors $\tau(n)$	<code>divisors(n), numdiv</code>
sum of (k -th powers of) divisors of n	<code>sigma($n, \{k\}$)</code>
Möbius μ -function	<code>moebius(x)</code>
Ramanujan's τ -function	<code>ramanujantau(x)</code>

Combinatorics

factorial of x	<code>x!</code> or <code>factorial(x)</code>
binomial coefficient $\binom{x}{k}$	<code>binomial($x, \{k\}$)</code>
Bernoulli number B_n as real/rational	<code>bernreal(n), bernfrac</code>
$[B_0, B_2, \dots B_{2k}]$	<code>bernvec(k)</code>
Bernoulli polynomial $B_n(x)$	<code>bernpol($n, \{x\}$)</code>
Euler numbers	<code>eulerfrac, eulerreal, eulervec</code>
Euler polynomial $E_n(x)$	<code>eulerpol($n, \{x\}$)</code>
Eulerian polynomial $A_n(x)$	<code>eulerianpol</code>
Fibonacci number F_n	<code>fibonacci(n)</code>
Harmonic number $H_{n,r} = 1^{-r} + \dots + n^{-r}$	<code>harmonic(n, r)</code>
Stirling numbers $s(n, k)$ and $S(n, k)$	<code>stirling($n, k, \{flag\}$)</code>

Pari-GP reference card

(PARI-GP version 2.15.3)

number of partitions of n	<code>numbpart(n)</code>
k -th permutation on n letters	<code>numtoperm(n, k)</code>
...index k of permutation v	<code>permtomv(v)</code>
order of permutation p	<code>permorder(p)</code>
signature of permutation p	<code>permsign(p)</code>
cyclic decomposition of permutation p	<code>permcycles(p)</code>

Multiplicative groups $(\mathbf{Z}/N\mathbf{Z})^*$, \mathbf{F}_q^*

Euler ϕ -function	<code>eulerphi(x)</code>
multiplicative order of x (divides ϕ)	<code>znorder($x, \{o\}$), fforder</code>
primitive root mod q / x .mod	<code>znprimroot(q), fprimroot(x)</code>
structure of $(\mathbf{Z}/n\mathbf{Z})^*$	<code>znstar(n)</code>
discrete logarithm of x in base g	<code>znlog($x, g, \{o\}$), fflag</code>
Kronecker-Legendre symbol $(\frac{x}{y})$	<code>kronecker(x, y)</code>
quadratic Hilbert symbol (at p)	<code>hilbert($x, y, \{p\}$)</code>

Euclidean algorithm, continued fractions

CRT: solve $z \equiv x$ and $z \equiv y$	<code>chinese(x, y)</code>
minimal u, v so $xu + yv = \gcd(x, y)$	<code>gcdext(x, y)</code>
half-gcd algorithm	<code>halfgcd(x, y)</code>
continued fraction of x	<code>confrac($x, \{b\}, \{lmax\}$)</code>
last convergent of continued fraction x	<code>confracpnqn(x)</code>
rational approximation to x (den. $\leq B$)	<code>bestappr($x, \{B\}$)</code>
recognize $x \in \mathbf{C}$ as polmod mod $T \in \mathbf{Z}[X]$	<code>bestapprnf(x, T)</code>

Miscellaneous

integer square / n -th root of x	<code>sqrtint(x), sqtrtnint(x, n)</code>
largest integer e s.t. $b^e \leq b$, $e = \lfloor \log_b(x) \rfloor$	<code>logint($x, b, \{\&z\}$)</code>

Characters

Let $cyc = [d_1, \dots, d_k]$ represent an abelian group $G = \oplus (\mathbf{Z}/d_j\mathbf{Z}) \cdot g_j$ or any structure G affording a .cyc method; e.g. `znstar($q, 1$)` for Dirichlet characters. A character χ is coded by $[c_1, \dots, c_k]$ such that $\chi(g_j) = e(n_j/d_j)$.
 $\chi \cdot \psi$; χ^{-1} ; $\chi \cdot \psi^{-1}$; χ^k `charmul, charconj, chardiv, charpow`
order of χ `charorder(cyc, χ)`
kernel of χ `charker(cyc, χ)`
 $\chi(x)$, G a GP group structure `chareval($G, \chi, x, \{z\}$)`
Galois orbits of characters `chargalois(G)`

Dirichlet Characters

initialize $G = (\mathbf{Z}/q\mathbf{Z})^*$	<code>G = znstar($q, 1$)</code>
convert datum D to $[G, \chi]$	<code>znchar(D)</code>
is χ odd?	<code>zncharisodd(G, χ)</code>
real $\chi \rightarrow$ Kronecker symbol (D/\cdot)	<code>znchartokronecker(G, χ)</code>
conductor of χ	<code>zncharconductor(G, χ)</code>
$[G_0, \chi_0]$ primitive attached to χ	<code>znchartoprimitive(G, χ)</code>
induce $\chi \in \hat{G}$ to $\mathbf{Z}/N\mathbf{Z}$	<code>zncharinduce(G, χ, N)</code>
χp	<code>znchardecompose(G, χ, p)</code>
$\prod_p (\chi, N) \chi p$	<code>znchardecompose(G, χ, Q)</code>
complex Gauss sum $G_a(\chi)$	<code>znchargauss(G, χ)</code>

Conrey labelling

Conrey label $m \in (\mathbf{Z}/q\mathbf{Z})^* \rightarrow$ character	<code>znconreychar(G, m)</code>
character \rightarrow Conrey label	<code>znconreyexp(G, χ)</code>
log on Conrey generators	<code>znconreylog(G, m)</code>
conductor of χ (χ_0 primitive)	<code>znconreyconductor($G, \chi, \{\chi_0\}$)</code>

True-False Tests

is x the disc. of a quadratic field?	<code>isfundamental(x)</code>
is x a prime?	<code>isprime(x)</code>
is x a strong pseudo-prime?	<code>ispseudoprime(x)</code>
is x square-free?	<code>issquarefree(x)</code>
is x a square?	<code>issquare($x, \{\&n\}$)</code>
is x a perfect power?	<code>ispower($x, \{k\}, \{\&n\}$)</code>
is x a perfect power of a prime? ($x = p^n$)	<code>isprimepower($x, \&n$)</code>
... of a pseudoprime?	<code>ispseudoprimepower($x, \&n$)</code>
is x powerful?	<code>ispowerful(x)</code>
is x a totient? ($x = \varphi(n)$)	<code>istotient($x, \{\&n\}$)</code>
is x a polygonal number? ($x = P(s, n)$)	<code>ispolygonal($x, s, \{\&n\}$)</code>
is pol irreducible?	<code>polisirreducible(pol)</code>

Graphic Functions

crude graph of $expr$ between a and b	<code>plot($X = a, b, expr$)</code>
High-resolution plot (immediate plot)	
plot $expr$ between a and b	<code>ploth($X = a, b, expr, \{flag\}, \{n\}$)</code>
plot points given by lists lx, ly	<code>plothraw($lx, ly, \{flag\}$)</code>
terminal dimensions	<code>plotsizes()</code>

Rectwindow functions

init window w , with size x, y	<code>plotinit(w, x, y)</code>
erase window w	<code>plotkill(w)</code>
copy w to w_2 with offset (dx, dy)	<code>plotcopy(w, w_2, dx, dy)</code>
slice contents of w	<code>plotclip(w)</code>
scale coordinates in w	<code>plotscale(w, x_1, x_2, y_1, y_2)</code>
ploth in w	<code>plotrecth($w, X = a, b, expr, \{flag\}, \{n\}$)</code>
plothraw in w	<code>plotrecthraw($w, data, \{flag\}$)</code>
draw window w_1 at $(x_1, y_1), \dots$	<code>plotdraw($[[w_1, x_1, y_1], \dots]$)</code>

Low-level Rectwindow Functions

set current drawing color in w to c	<code>plotcolor(w, c)</code>
current position of cursor in w	<code>plotcursor(w)</code>
write s at cursor's position	<code>plotstring(w, s)</code>
move cursor to (x, y)	<code>plotmove(w, x, y)</code>
move cursor to $(x + dx, y + dy)$	<code>plotrmove(w, dx, dy)</code>
draw a box to (x_2, y_2)	<code>plotbox(w, x_2, y_2)</code>
draw a box to $(x + dx, y + dy)$	<code>plotrbox(w, dx, dy)</code>
draw polygon	<code>plotlines($w, lx, ly, \{flag\}$)</code>
draw points	<code>plotpoints(w, lx, ly)</code>
draw line to $(x + dx, y + dy)$	<code>plotrline(w, dx, dy)</code>
draw point $(x + dx, y + dy)$	<code>plotrpoint(w, dx, dy)</code>

Convert to Postscript or Scalable Vector Graphics

The format f is either "ps" or "svg".	
as ploth	<code>plotlexport($f, X = a, b, expr, \{flag\}, \{n\}$)</code>
as plothraw	<code>plotlrawexport($f, lx, ly, \{flag\}$)</code>
as plotdraw	<code>plotexport($f, [[w_1, x_1, y_1], \dots]$)</code>

Based on an earlier version by Joseph H. Silverman
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