Pari-GP reference card

(PARI-GP version 2.15.3)

Note: optional arguments are surrounded by braces {}.
To start the calculator, type its name in the terminal: `gp`
To exit `gp`, type `quit`, `\q`, or `<C-D>` at prompt.

Help

describe function	`?`*function*
extended description	`??`*keyword*
list of relevant help topics	`???`*pattern*
name of GP-1.39 function f in GP-2.*	`whatnow(`f`)`

Input/Output

previous result, the result before	`%`, `%\``, `%\`\``, etc.
n-th result since startup	`%`n
separate multiple statements on line	`;`
extend statement on additional lines	`\`
extend statements on several lines	`{`seq_1`; `seq_2`;}`
comment	`/* ... */`
one-line comment, rest of line ignored	`\\ ...`

Metacommands & Defaults

set default d to val	`default(`$\{d\},\{val\}$`)`
toggle timer on/off	`#`
print time for last result	`##`
print defaults	`\d`
set debug level to n	`\g` n
set memory debug level to n	`\gm` n
set n significant digits / bits	`\p` n, `\pb` n
set n terms in series	`\ps` n
quit GP	`\q`
print the list of PARI types	`\t`
print the list of user-defined functions	`\u`
read file into GP	`\r` *filename*
set debuglevel for domain D to n	`setdebug(`D,n`)`

Debugger / break loop

get out of break loop	`break` or `<C-D>`
go up/down n frames	`dbg_up(`$\{n\}$`)`, `dbg_down`
set break point	`breakpoint()`
examine object o	`dbg_x(`o`)`
current error data	`dbg_err()`
number of objects on heap and their size	`getheap()`
total size of objects on PARI stack	`getstack()`

PARI Types & Input Formats

`t_INT`. Integers; hex, binary	± 31; $\pm$`0x1F`, $\pm$`0b101`
`t_REAL`. Reals	± 3.14, 6.022 E23
`t_INTMOD`. Integers modulo m	`Mod(`n,m`)`
`t_FRAC`. Rational Numbers	n/m
`t_FFELT`. Elt in finite field $\mathbf{F}_q$	`ffgen(`q`,'t)`
`t_COMPLEX`. Complex Numbers	$x + y*$ `I`
`t_PADIC`. p-adic Numbers	$x + O(p\hat{}k)$
`t_QUAD`. Quadratic Numbers	$x + y*$ `quadgen(D,{'w})`
`t_POLMOD`. Polynomials modulo g	`Mod(`f,g`)`
`t_POL`. Polynomials	$a*x\hat{}n + \cdots + b$
`t_SER`. Power Series	$f + O(x\hat{}k)$
`t_RFRAC`. Rational Functions	f/g
`t_QFB`. Binary quadratic form	`Qfb(`a,b,c`)`
`t_VEC`/`t_COL`. Row/Column Vectors	`[`x,y,z`]`, `[`x,y,z`]~`
`t_VEC` integer range	`[1..10]`
`t_VECSMALL`. Vector of small ints	`Vecsmall([`x,y,z`])`
`t_MAT`. Matrices	`[`a,b`;`c,d`]`
`t_LIST`. Lists	`List([`x,y,z`])`
`t_STR`. Strings	`"abc"`
`t_INFINITY`. $\pm\infty$	`+oo, -oo`

Reserved Variable Names

$\pi \approx 3.14$, $\gamma \approx 0.57$, $C \approx 0.91$, $I = \sqrt{-1}$	`Pi`, `Euler`, `Catalan`, `I`
Landau's big-oh notation	`O`

Information about an Object, Precision

PARI type of object x	`type(`x`)`
length of x / size of x in memory	`#`x, `sizebyte(`x`)`
real precision / bit precision of x	`precision(`x`)`, `bitprecision(`x`)`
p-adic, series prec. of x	`padicprec(`x,p`)`, `serprec(`x,v`)`
current dynamic precision	`getlocalprec`, `getlocalbitprec`

Operators

basic operations	`+`, `-` , `*`, `/`, `^`, `sqr`
i←i+1, i←i-1, i←i*j, ...	`i++`, `i--`, `i*=j`,...
Euclidean quotient, remainder	x`\/`y, x`\`y, x`%`y, `divrem(`x,y`)`
shift x left or right n bits	x`<<`n, x`>>`n or `shift(`$x,\pm n$`)`
multiply by 2^n	`shiftmul(`x,n`)`
comparison operators	`<=`, `<`, `>=`, `>`, `==`, `!=`, `===`, `lex`, `cmp`
boolean operators (or, and, not)	`||`, `&&`, `!`
bit operations	`bitand`, `bitneg`, `bitor`, `bitxor`, `bitnegimply`
maximum/minimum of x and y	`max(`x,y`)`, `min(`x,y`)`
sign of x (gives $-1, 0, 1$)	`sign(`x`)`
binary exponent of x	`exponent(`x`)`
derivative of f, 2nd derivative, etc.	f`'`, f`''`, ...
differential operator	`diffop(`$f,v,d,\{n=1\}$`)`
quote operator (formal variable)	`'x`
assignment	`x =` *value*
simultaneous assignment $x \leftarrow v[1]$, $y \leftarrow v[2]$	`[x,y] = v`

Select Components

Caveat: components start at index $n = 1$.

n-th component of x	`component(`x,n`)`
n-th component of vector/list x	x`[`n`]`
components $a, a+1, \ldots, b$ of vector x	x`[`a`..`b`]`
(m,n)-th component of matrix x	x`[`m,n`]`
row m or column n of matrix x	x`[`m`,]`, x`[,`n`]`
numerator/denominator of x	`numerator(`x`)`, `denominator(`x`)`

Random Numbers

random integer/prime in $[0, N[$	`random(`N`)`, `randomprime(`N`)`
get/set random seed	`getrand`, `setrand(`s`)`

Conversions

to vector, matrix, vec. of small ints	`Col/Vec`,`Mat`,`Vecsmall`
to list, set, map, string	`List`, `Set`, `Map`, `Str`
create (x mod y)	`Mod(`x,y`)`
make x a polynomial of v	`Pol(`$x,\{v\}$`)`
variants of `Pol` *et al.*, in reverse order	`Polrev`, `Vecrev`, `Colrev`
make x a power series of v	`Ser(`$x,\{v\}$`)`
convert x to simplest possible type	`simplify(`x`)`
object x with real precision n	`precision(`x,n`)`
object x with bit precision n	`bitprecision(`x,n`)`
set precision to p digits in dynamic scope	`localprec(`p`)`
set precision to p bits in dynamic scope	`localbitprec(`p`)`

Character strings

convert to TeX representation	`strtex(`x`)`
string from bytes / from format+args	`strchr`, `strprintf`
split string / join strings	`strsplit`, `strjoin`
convert time t ms. to h, m, s, ms format	`strtime(`t`)`

Conjugates and Lifts

conjugate of a number x	`conj(`x`)`
norm of x, product with conjugate	`norm(`x`)`
L^p norm of x (L^∞ if no p)	`normlp(`$x,\{p\}$`)`
square of L^2 norm of x	`norml2(`x`)`
lift of x from Mods and p-adics	`lift`, `centerlift(`x`)`
recursive lift	`liftall`
lift all `t_INT` and `t_PADIC` (→`t_INT`)	`liftint`
lift all `t_POLMOD` (→`t_POL`)	`liftpol`

Lists, Sets & Maps

Sets (= row vector with strictly increasing entries w.r.t. `cmp`)

intersection of sets x and y	`setintersect(`x,y`)`
set of elements in x not belonging to y	`setminus(`x,y`)`
symmetric difference $x \Delta y$	`setdelta(`x,y`)`
union of sets x and y	`setunion(`x,y`)`
does y belong to the set x	`setsearch(`$x,y,\{flag\}$`)`
set of all $f(x,y)$, $x \in X$, $y \in Y$	`setbinop(`f,X,Y`)`
is x a set ?	`setisset(`x`)`

Lists. create empty list: $L =$ `List()`

append x to list L	`listput(`$L,x,\{i\}$`)`
remove i-th component from list L	`listpop(`$L,\{i\}$`)`
insert x in list L at position i	`listinsert(`L,x,i`)`
sort the list L in place	`listsort(`$L,\{flag\}$`)`

Maps. create empty dictionary: $M =$ `Map()`

attach value v to key k	`mapput(`M,k,v`)`
recover value attach to key k or error	`mapget(`M,k`)`
is key k in the dict? (set v to $M(k)$)	`mapisdefined(`$M,k,\{\&v\}$`)`
remove k from map domain	`mapdelete(`M,k`)`

GP Programming

User functions and closures

x, y are formal parameters; y defaults to `Pi` if parameter omitted;
z, t are local variables (lexical scope), z initialized to 1.

`fun(x, y=Pi) = my(z=1, t);` *seq*
`fun = (x, y=Pi) -> my(z=1, t);` *seq*

attach help message h to s	`addhelp(`s,h`)`
undefine symbol s (also kills help)	`kill(`s`)`

Control Statements (X: formal parameter in expression *seq*)

if $a \neq 0$, evaluate seq_1, else seq_2	`if(`$a,\{seq_1\},\{seq_2\}$`)`
eval. *seq* for $a \leq X \leq b$	`for(`$X=a,b,seq$`)`
... for $X \in v$	`foreach(`v,X,seq`)`
... for primes $a \leq X \leq b$	`forprime(`$X=a,b,seq$`)`
... for primes $\equiv a \pmod q$	`forprimestep(`$X=a,b,q,seq$`)`
... for composites $a \leq X \leq b$	`forcomposite(`$X=a,b,seq$`)`
... for $a \leq X \leq b$ stepping s	`forstep(`$X=a,b,s,seq$`)`
... for X dividing n	`fordiv(`n,X,seq`)`
... $X = [n, factor(n)]$, $a \leq n \leq b$	`forfactored(`$X=a,b,seq$`)`
... as above, n squarefree	`forsquarefree(`$X=a,b,seq$`)`
... $X = [d, factor(d)]$, $d \mid n$	`fordivfactored(`n,X,seq`)`
multivariable `for`, lex ordering	`forvec(`$X=v,seq$`)`

© 2022 Karim Belabas. Permissions on back. v2.38

Pari-GP reference card

(PARI-GP version 2.15.3)

loop over partitions of n	`forpart`$(p=n, seq)$
... permutations of S	`forperm`(S, p, seq)
... subsets of $\{1,\dots,n\}$	`forsubset`(n, p, seq)
... k-subsets of $\{1,\dots,n\}$	`forsubset`$([n,k], p, seq)$
... vectors v, $q(v) \le B$; $q > 0$	`forqfvec`(v, q, b, seq)
... $H < G$ finite abelian group	`forsubgroup`$(H=G)$
evaluate seq until $a \neq 0$	`until`(a, seq)
while $a \neq 0$, evaluate seq	`while`(a, seq)
exit n innermost enclosing loops	`break`$(\{n\})$
start new iteration of n-th enclosing loop	`next`$(\{n\})$
return x from current subroutine	`return`$(\{x\})$

Exceptions, warnings

raise an exception / warning	`error(), warning()`
type of error message E	`errname`(E)
try seq_1, evaluate seq_2 on error	`iferr`(seq_1, E, seq_2)

Functions with closure arguments / results

number of arguments of f	`arity`(f)
select from v according to f	`select`(f, v)
apply f to all entries in v	`apply`(f, v)
evaluate $f(a_1,\dots,a_n)$	`call`(f, a)
evaluate $f(\dots f(f(a_1,a_2),a_3)\dots,a_n)$	`fold`(f, a)
calling function as closure	`self()`

Sums & Products

sum $X=a$ to $X=b$, initialized at x	`sum`$(X=a,b,expr,\{x\})$
sum entries of vector v	`vecsum`(v)
product of all vector entries	`vecprod`(v)
sum $expr$ over divisors of n	`sumdiv`$(n, X, expr)$
... assuming $expr$ multiplicative	`sumdivmult`$(n, X, expr)$
product $a \le X \le b$, initialized at x	`prod`$(X=a,b,expr,\{x\})$
product over primes $a \le X \le b$	`prodeuler`$(X=a,b,expr)$

Sorting

sort x by k-th component	`vecsort`$(x, \{k\}, \{fl=0\})$
min. m of x ($m = x[i]$), max.	`vecmin`$(x, \{\&i\})$, `vecmax`
does y belong to x, sorted wrt. f	`vecsearch`$(x, y, \{f\})$
$\prod g^x \to$ factorization ($\Rightarrow$ sorted, unique g)	`matreduce`(m)

Input/Output

print with/without `\n`, TEX format	`print`, `print1`, `printtex`
pretty print matrix	`printp`
print fields with separator	`printsep`$(sep,\dots)$`,, printsep1`
formatted printing	`printf()`
write $args$ to file	`write`, `write1`, `writetex`$(file, args)$
write x in binary format	`writebin`$(file, x)$
read file into GP	`read`$(\{file\})$
... return as vector of lines	`readvec`$(\{file\})$
... return as vector of strings	`readstr`$(\{file\})$
read a string from keyboard	`input()`

Files and file descriptors

File descriptors allow efficient small consecutive reads or writes from or to a given file. The argument n below is always a descriptor, attached to a file in `r`(ead), `w`(rite) or `a`(ppend) mode.

get descriptor n for file $path$ in given $mode$	`fileopen`$(path, mode)$
... from shell cmd output (pipe)	`fileextern`(cmd)
close descriptor	`fileclose`(n)
commit pending write operations	`fileflush`(n)
read logical line from file	`fileread`(n)
... raw line from file	`filereadstr`(n)
write s`\n` to file	`filewrite`(n, s)
... write s to file	`filewrite1`(n, s)

Timers

CPU time in ms and reset timer	`gettime()`
CPU time in ms since gp startup	`getabstime()`
time in ms since UNIX Epoch	`getwalltime()`
timeout command after s seconds	`alarm`$(s, expr)$

Interface with system

allocates a new stack of s bytes	`allocatemem`$(\{s\})$
alias old to new	`alias`(new, old)
install function from library	`install`$(f, code, \{gpf\}, \{lib\})$
execute system command a	`system`(a)
... and feed result to GP	`extern`(a)
... returning GP string	`externstr`(a)
get `$VAR` from environment	`getenv("VAR")`
expand env. variable in string	`strexpand`(x)

Parallel evaluation

These functions evaluate their arguments in parallel (pthreads or MPI); args. must not access global variables (use `export` for this) and must be free of side effects. Enabled if threading engine is not *single* in gp header.

evaluate f on $x[1],\dots,x[n]$	`parapply`(f, x)
evaluate closures $f[1],\dots,f[n]$	`pareval`(f)
as `select`	`parselect`$(f, A, \{flag\})$
as `sum`	`parsum`$(i=a,b,expr)$
as `vector`	`parvector`$(n, i, \{expr\})$
eval f for $i=a,\dots,b$	`parfor`$(i=a,\{b\},f,\{r\},\{f_2\})$
... for each element x in v	`parforeach`$(v,x,f,\{r\},\{f_2\})$
... for p prime in $[a,b]$	`parforprime`$(p=a,\{b\},f,\{r\},\{f_2\})$
... for $p=a \bmod q$	`parforprimestep`$(p=a,\{b\},q,f,\{r\},\{f_2\})$
... multivariate	`parforvec`$(X=v,f,\{r\},\{f_2\},\{flag\})$
export x to parallel world	`export`(x)
... all dynamic variables	`exportall()`
frees exported value x	`unexport`(x)
... all exported values	`unexportall()`

Linear Algebra

dimensions of matrix x	`matsize`(x)
multiply two matrices	x `*` y
... assuming result is diagonal	`matmultodiagonal`(x, y)
concatenation of x and y	`concat`$(x, \{y\})$
extract components of x	`vecextract`$(x, y, \{z\})$
transpose of vector or matrix x	x`~, mattranspose`(x)
adjoint of the matrix x	`matadjoint`(x)
eigenvectors/values of matrix x	`mateigen`(x)
characteristic/minimal polynomial of x	`charpoly`(x), `minpoly`(x)
trace/determinant of matrix x	`trace`(x), `matdet`(x)
permanent of matrix x	`matpermanent`(x)
Frobenius form of x	`matfrobenius`(x)
QR decomposition	`matqr`(x)
apply `matqr`'s transform to v	`mathouseholder`(Q, v)

Constructors & Special Matrices

$\{g(x)\colon x \in v \text{ s.t. } f(x)\}$	`[g(x) | x <- v, f(x)]`
$\{x\colon x \in v \text{ s.t. } f(x)\}$	`[x | x <- v, f(x)]`
$\{g(x)\colon x \in v\}$	`[g(x) | x <- v]`
row vec. of $expr$ eval'ed at $1 \le i \le n$	`vector`$(n, \{i\}, \{expr\})$
col. vec. of $expr$ eval'ed at $1 \le i \le n$	`vectorv`$(n, \{i\}, \{expr\})$
vector of small ints	`vectorsmall`$(n, \{i\}, \{expr\})$
$[c, c\cdot x, \dots, c\cdot x^n]$	`powers`$(x, n, \{c=1\})$
$[1, 2^x, \dots, n^x]$	`dirpowers`(n, x)
matrix $1 \le i \le m$, $1 \le j \le n$	`matrix`$(m, n, \{i\}, \{j\}, \{expr\})$
define matrix by blocks	`matconcat`(B)
diagonal matrix with diagonal x	`matdiagonal`(x)
is x diagonal?	`matisdiagonal`(x)
$x \cdot$ `matdiagonal`(d)	`matmuldiagonal`(x, d)
$n \times n$ identity matrix	`matid`(n)
Hessenberg form of square matrix x	`mathess`(x)
$n \times n$ Hilbert matrix $H_{ij} = (i+j-1)^{-1}$	`mathilbert`(n)
$n \times n$ Pascal triangle	`matpascal`$(n-1)$
companion matrix to polynomial x	`matcompanion`(x)
Sylvester matrix of x and y	`polsylvestermatrix`(x, y)

Gaussian elimination

kernel of matrix x	`matker`$(x, \{flag\})$
intersection of column spaces of x and y	`matintersect`(x, y)
solve $MX = B$ (M invertible)	`matsolve`(M, B)
one sol of $M * X = B$	`matinverseimage`(M, B)
basis for image of matrix x	`matimage`(x)
columns of x *not* in `matimage`	`matimagecompl`(x)
supplement columns of x to get basis	`matsupplement`(x)
rows, cols to extract invertible matrix	`matindexrank`(x)
rank of the matrix x	`matrank`(x)
solve $MX = B \bmod D$	`matsolvemod`(M, D, B)
image mod D	`matimagemod`(M, D)
kernel mod D	`matkermod`(M, D)
inverse mod D	`matinvmod`(M, D)
determinant mod D	`matdetmod`(M, D)

Lattices & Quadratic Forms

Quadratic forms

evaluate ${}^t xQy$	`qfeval`$(\{Q=id\}, x, y)$
evaluate ${}^t xQx$	`qfeval`$(\{Q=id\}, x)$
signature of quad form ${}^t y * x * y$	`qfsign`(x)
decomp into squares of ${}^t y * x * y$	`qfgaussred`(x)
eigenvalues/vectors for real symmetric x	`qfjacobi`(x)

HNF and SNF

upper triangular Hermite Normal Form	`mathnf`(x)
HNF of x where d is a multiple of $\det(x)$	`mathnfmod`(x, d)
multiple of $\det(x)$	`matdetint`(x)
HNF of $(x \mid$ `diagonal`$(D))$	`mathnfmodid`(x, D)
elementary divisors of x	`matsnf`(x)
q-rank from elementary divisors	`snfrank`(v, q)
elementary divisors of $\mathbf{Z}[a]/(f'(a))$	`poldiscreduced`(f)
integer kernel of x	`matkerint`(x)
$\mathbf{Z}$-module $\leftrightarrow$ $\mathbf{Q}$-vector space	`matrixqz`(x, p)

Lattices

LLL-algorithm applied to columns of x	`qflll`$(x, \{flag\})$
... for Gram matrix of lattice	`qflllgram`$(x, \{flag\})$
find up to m sols of `qfnorm`$(x, y) \le b$	`qfminim`(x, b, m)
v, $v[i]$:=number of y s.t. `qfnorm`$(x, y) = i$	`qfrep`$(x, B, \{flag\})$
perfection rank of x	`qfperfection`(x)
find isomorphism between q and Q	`qfisom`(q, Q)
precompute for isomorphism test with q	`qfisominit`(q)
automorphism group of q	`qfauto`(q)

Based on an earlier version by Joseph H. Silverman

November 2022 v2.38. Copyright © 2022 K. Belabas
Permission is granted to make and distribute copies of this card provided the copyright and this permission notice are preserved on all copies.

Send comments and corrections to ⟨Karim.Belabas@math.u-bordeaux.fr⟩

convert qfauto for GAP/Magma — qfautoexport($G, \{flag\}$)
orbits of V under $G \subset \mathrm{GL}(V)$ — qforbits(G, V)

Polynomials & Rational Functions

all defined polynomial variables — variables()
get var. of highest priority (higher than v) — varhigher($name, \{v\}$)
... of lowest priority (lower than v) — varlower($name, \{v\}$)

Coefficients, variables and basic operators

degree of f — poldegree(f)
coef. of degree n of f, leading coef. — polcoef(f, n), pollead
main variable / all variables in f — variable(f), variables(f)
replace x by y in f — subst(f, x, y)
evaluate f replacing vars by their value — eval(f)
replace polynomial expr. $T(x)$ by y in f — substpol(f, T, y)
replace $x_1, \dots, x_n$ by $y_1, \dots, y_n$ in f — substvec(f, x, y)
$f \in A[x]$; reciprocal polynomial $x^{\deg f} f\left(\frac{1}{x}\right)$ — polrecip(f)
gcd of coefficients of f — content(f)
derivative of f w.r.t. x — deriv($f, \{x\}$)
... n-th derivative of f — derivn($f, n, \{x\}$)
formal integral of f w.r.t. x — intformal($f, \{x\}$)
formal sum of f w.r.t. x — sumformal($f, \{x\}$)

Constructors & Special Polynomials

interpolation polynomial at $(x[1], y[1]), \dots, (x[n], y[n])$, evaluated at t, with error estimate e — polinterpolate($x, \{y\}, \{t\}, \{\&e\}$)
T_n/U_n, H_n — polchebyshev(n), polhermite(n)
P_n, $L_n^{(\alpha)}$ — pollegendre(n), pollaguerre(n, a)
n-th cyclotomic polynomial Φ_n — polcyclo(n)
return n if $f = \Phi_n$, else 0 — poliscyclo(f)
is f a product of cyclotomic polynomials? — poliscycloprod(f)
Zagier's polynomial of index (n, m) — polzagier(n, m)

Resultant, elimination

discriminant of polynomial f — poldisc(f)
find factors of poldisc(f) — poldiscfactors(f)
resultant $R = \mathrm{Res}_v(f, g)$ — polresultant($f, g, \{v\}$)
$[u, v, R]$, $xu + yv = \mathrm{Res}_v(f, g)$ — polresultantext($x, y, \{v\}$)
solve Thue equation $f(x, y) = a$ — thue($t, a, \{sol\}$)
initialize t for Thue equation solver — thueinit(f)

Roots and Factorization (Complex/Real)

complex roots of f — polroots(f)
bound complex roots of f — polrootsbound(f)
number of real roots of f (in $[a, b]$) — polsturm($f, \{[a, b]\}$)
real roots of f (in $[a, b]$) — polrootsreal($f, \{[a, b]\}$)
complex embeddings of t_POLMOD z — conjvec(z)

Roots and Factorization (Finite fields)

factor f mod p, roots — factormod(f, p), polrootsmod
factor f over $\mathbf{F}_p[x]/(T)$, roots — factormod($f, [T, p]$), polrootsmod
squarefree factorization of f in $\mathbf{F}_q[x]$ — factormodSQF($f, \{D\}$)
distinct degree factorization of f in $\mathbf{F}_q[x]$ — factormodDDF($f, \{D\}$)
factor n-th cyclotomic pol. Φ_n mod p — factormodcyclo(n, p)

Roots and Factorization (p-adic fields)

factor f over $\mathbf{Q}_p$, roots — factorpadic(f, p, r), polrootspadic
p-adic root of f congruent to a mod p — padicappr(f, a)
Newton polygon of f for prime p — newtonpoly(f, p)
Hensel lift $A/\mathrm{lc}(A) = \prod_i B[i] \bmod p^e$ — polhensellift(A, B, p, e)
$T = \prod (x - z_i) \mapsto \prod (x - \omega(z_i))] \in \mathbf{Z}_p[x]$ — polteichmuller(T, p, e)
extensions of $\mathbf{Q}_p$ of degree N — padicfields(p, N)

© 2022 Karim Belabas. Permissions on back. v2.38

Pari-GP reference card

(PARI-GP version 2.15.3)

Roots and Factorization (Miscellaneous)

symmetric powers of roots of f up to n — polsym(f, n)
Graeffe transform of f, $g(x^2) = f(x)f(-x)$ — polgraeffe(f)
factor f over coefficient field — factor(f)
cyclotomic factors of $f \in \mathbf{Q}[X]$ — polcyclofactors(f)

Finite Fields

A finite field is encoded by any element (t_FFELT).

find irreducible $T \in \mathbf{F}_p[x]$, $\deg T = n$ — ffinit($p, n, \{x\}$)
Create t in $\mathbf{F}_q \simeq \mathbf{F}_p[t]/(T)$ — t = ffgen(T,'t)
... indirectly, with implicit T — t = ffgen(q,'t); T = t.mod
map m from $\mathbf{F}_q \ni a$ to $\mathbf{F}_{q^k} \ni b$ — m = ffembed(a, b)
build $K = \mathbf{F}_q[x]/(P)$ extending $\mathbf{F}_q \ni a$, — ffextend(a, P)
evaluate map m on x — ffmap(m, x)
inverse map of m — ffinvmap(m)
compose maps $m \circ n$ — ffcompomap(m, n)
x as polmod over codomain of map m — ffmaprel(m, x)
F^n over $\mathbf{F}_q \ni a$ — fffrobenius(a, n)
$\#\{\text{monic irred. } T \in \mathbf{F}_q[x], \deg T = n\}$ — ffnbirred(q, n)

Formal & p-adic Series

truncate power series or p-adic number — truncate(x)
valuation of x at p — valuation(x, p)

Dirichlet and Power Series

Taylor expansion around 0 of f w.r.t. x — taylor(f, x)
Laurent series of closure F up to x^k — laurentseries(f, k)
$\sum a_k b_k t^k$ from $\sum a_k t^k$ and $\sum b_k t^k$ — serconvol(a, b)
$f = \sum a_k t^k$ from $\sum (a_k/k!) t^k$ — serlaplace(f)
reverse power series F so $F(f(x)) = x$ — serreverse(f)
remove terms of degree $< n$ in f — serchop(f, n)
Dirichlet series multiplication / division — dirmul, dirdiv(x, y)
Dirichlet Euler product (b terms) — direuler($p = a, b, expr$)

Transcendental and p-adic Functions

real, imaginary part of x — real(x), imag(x)
absolute value, argument of x — abs(x), arg(x)
square/nth root of x — sqrt(x), sqrtn($x, n, \{\&z\}$)
all n-th roots of 1 — rootsof1(n)
FFT of $[f_0, \dots, f_{n-1}]$ — w = fftinit(n), fft/fftinv(w, f)
trig functions — sin, cos, tan, cotan, sinc
inverse trig functions — asin, acos, atan
hyperbolic functions — sinh, cosh, tanh, cotanh
inverse hyperbolic functions — asinh, acosh, atanh
$\log(x)$, $\log(1 + x)$, e^x, $e^x - 1$ — log, log1p, exp, expm1
Euler Γ function, $\log \Gamma$, Γ'/Γ — gamma, lngamma, psi
half-integer gamma function $\Gamma(n + 1/2)$ — gammah(n)
Riemann's zeta $\zeta(s) = \sum n^{-s}$ — zeta(s)
$\sum_{1 \le n \le N} n^s$ — dirpowerssum(N, s)
Hurwitz's $\zeta(s, x) = \sum (n + x)^{-s}$ — zetahurwitz(s, x)
Lerch $\Phi(z, s, x) = \sum z^n (n + x)^{-s}$ — lerchphi(z, s, x)
Lerch $L(s, x, t) = \Phi(e^{2i\pi t}, s, x)$ — lerchzeta(s, x, t)
multiple zeta value (MZV), $\zeta(s_1, \dots, s_k)$ — zetamult($s, \{T\}$)
all MZVs for weight $\sum s_i = n$ — zetamultall(n)
convert MZV id to $[s_1, \dots, s_k]$ — zetamultconvert($f, \{flag\}$)
MZV dual sequence — zetamultdual(s)
multiple polylog $Li_{s_1, \dots s_k}(z_1, \dots, z_k)$ — polylogmult(s, z)

incomplete Γ function ($y = \Gamma(s)$) — incgam($s, x, \{y\}$)
complementary incomplete Γ — incgamc(s, x)
$\int_x^\infty e^{-t}\, dt/t$, $(2/\sqrt{\pi}) \int_x^\infty e^{-t^2} dt$ — eint1, erfc
elliptic integral of 1st and 2nd kind — ellK(k), ellE(k)
dilogarithm of x — dilog(x)
m-th polylogarithm of x — polylog($m, x, \{flag\}$)
U-confluent hypergeometric function — hyperu(a, b, u)
Hypergeometric ${}_pF_q(A, B; z)$ — hypergeom(A, B, z)
Bessel $J_n(x)$, $J_{n+1/2}(x)$ — besselj(n, x), besseljh(n, x)
Bessel I_ν, K_ν, H_ν^1, H_ν^2, Y_ν — (bessel)i, k, h1, h2, y
k-th zero of $J_\nu(x)$ — besseljzero($n\nu, \{k = 1\}$)
k-th zero of $Y_\nu(x)$ — besselyzero($n\nu, \{k = 1\}$)
Airy functions $A_i(x)$, $B_i(x)$ — airy(x)
Lambert W: x s.t. $xe^x = y$ — lambertw(y)
Teichmuller character of p-adic x — teichmuller(x)

Iterations, Sums & Products

Numerical integration for meromorphic functions

Behaviour at endpoint for Double Exponential (DE) methods: either a scalar ($a \in \mathbf{C}$, regular) or $\pm$oo (decreasing at least as x^{-2}) or

$(x - a)^{-\alpha}$ singularity — $[a, \alpha]$
exponential decrease $e^{-\alpha|x|}$ — $[\pm\infty, \alpha]$, $\alpha > 0$
slow decrease $|x|^\alpha$ — ... $\alpha < -1$
oscillating as $\cos(kx)$) — $\alpha = k$I, $k > 0$
oscillating as $\sin(kx)$) — $\alpha = -k$I, $k > 0$

numerical integration — intnum($x = a, b, f, \{T\}$)
weights T for intnum — intnuminit($a, b, \{m\}$)
weights T incl. kernel K — intfuncinit($t = a, b, K, \{m\}$)
integrate $(2i\pi)^{-1} f$ on circle $|z - a| = R$ — intcirc($x = a, R, f, \{T\}$)

Other integration methods

n-point Gauss-Legendre — intnumgauss($x = a, b, f, \{n\}$)
weights for n-point Gauss-Legendre — intnumgaussinit($\{n\}$)
quasi-periodic function, period $2H$ — intnumosc($x = a, f, H$)
Romberg (low accuracy) — intnumromb($x = a, b, f, \{flag\}$)

Numerical summation

sum of series $f(n)$, $n \ge a$ (low accuracy) — suminf($n = a, expr$)
sum of alternating/positive series — sumalt, sumpos
sum of series using Euler-Maclaurin — sumnum($n = a, f, \{T\}$)
... Sidi summation — sumnumsidi($n = a, f$)
$\sum_{n \ge a} F(n)$, F rational function — sumnumrat(F, a)
$\dots \sum_{p \ge a} F(p^s)$ — sumeulerrat($F, \{s = 1\}, \{a = 2\}$)
weights for sumnum, a as in DE — sumnuminit($\{\infty, a\}$)
sum of series by Monien summation — sumnummonien($n = a, f, \{T\}$)
weights for sumnummonien — sumnummonieninit($\{\infty, a\}$)
sum of series using Abel-Plana — sumnumap($n = a, f, \{T\}$)
weights for sumnumap, a as in DE — sumnumapinit($\{\infty, a\}$)
sum of series using Lagrange — sumnumlagrange($n = a, f, \{T\}$)
weights for sumnumlagrange — sumnumlagrangeinit

Products

product $a \le X \le b$, initialized at x — prod($X = a, b, expr, \{x\}$)
product over primes $a \le X \le b$ — prodeuler($X = a, b, expr$)
infinite product $a \le X \le \infty$ — prodinf($X = a, expr$)
$\prod_{n \ge a} F(n)$, F rational function — prodnumrat(F, a)
$\prod_{p \ge a} F(p^s)$ — prodeulerrat($F, \{s = 1\}, \{a = 2\}$)

Pari-GP reference card

(PARI-GP version 2.15.3)

Other numerical methods

real root of f in $[a,b]$; bracketed root	`solve`$(X=a,b,f)$
... interval splitting, step s	`solvestep`$(X=a,b,s,f,\{flag=0\})$
limit of $f(t)$, $t\to\infty$	`limitnum(f, {alpha})`
asymptotic expansion of f (rational)	`asympnum(f, {alpha})`
... $N+1$ terms as floats	`asympnumraw(f, N, {alpha})`
numerical derivation w.r.t x: $f'(a)$	`derivnum`$(x=a,f)$
evaluate continued fraction F at t	`contfraceval`$(F,t,\{L\})$
power series to cont. fraction (L terms)	`contfracinit`$(S,\{L\})$
Padé approximant (deg. denom. $\le B$)	`bestapprPade`$(S,\{B\})$

Elementary Arithmetic Functions

vector of binary digits of $\lvert x\rvert$	`binary`(x)
bit number n of integer x	`bittest`(x,n)
Hamming weight of integer x	`hammingweight`(x)
digits of integer x in base B	`digits`$(x,\{B=10\})$
sum of digits of integer x in base B	`sumdigits`$(x,\{B=10\})$
integer from digits	`fromdigits`$(v,\{B=10\})$
ceiling/floor/fractional part	`ceil`, `floor`, `frac`
round x to nearest integer	`round`$(x,\{\&e\})$
truncate x	`truncate`$(x,\{\&e\})$
gcd/LCM of x and y	`gcd`(x,y), `lcm`(x,y)
gcd of entries of a vector/matrix	`content`(x)

Primes and Factorization

extra prime table	`addprimes`$()$
add primes in v to prime table	`addprimes`(v)
remove primes from prime table	`removeprimes`(v)
Chebyshev $\pi(x)$, n-th prime p_n	`primepi`(x), `prime`(n)
vector of first n primes	`primes`(n)
smallest prime $\ge x$	`nextprime`(x)
largest prime $\le x$	`precprime`(x)
factorization of x	`factor`$(x,\{lim\})$
... selecting specific algorithms	`factorint`$(x,\{flag=0\})$
$n=df^2$, d squarefree/fundamental	`core`$(n,\{fl\})$, `coredisc`
certificate for (prime) N	`primecert`(N)
verifies a certificate c	`primecertisvalid`(c)
convert certificate to Magma/PRIMO	`primecertexport`
recover x from its factorization	`factorback`$(f,\{e\})$
$x\in\mathbf{Z}$, $\lvert x\rvert\le X$, $\gcd(N,P(x))\ge N$	`znchoppersmith`$(P,N,X,\{B\})$
divisors of N in residue class r mod s	`divisorslenstra`(N,r,s)

Divisors and multiplicative functions

number of prime divisors $\omega(n)$ / $\Omega(n)$	`omega`(n), `bigomega`
divisors of n / number of divisors $\tau(n)$	`divisors`(n), `numdiv`
sum of (k-th powers of) divisors of n	`sigma`$(n,\{k\})$
Möbius μ-function	`moebius`(x)
Ramanujan's τ-function	`ramanujantau`(x)

Combinatorics

factorial of x	x`!` or `factorial`(x)
binomial coefficient $\binom{x}{k}$	`binomial`$(x,\{k\})$
Bernoulli number B_n as real/rational	`bernreal`(n), `bernfrac`
$[B_0,B_2,\dots B_{2k}]$	`bernvec`(k)
Bernoulli polynomial $B_n(x)$	`bernpol`$(n,\{x\})$
Euler numbers	`eulerfrac`, `eulerreal`, `eulervec`
Euler polynomial $E_n(x)$	`eulerpol`$(n,\{x\})$
Eulerian polynomial $A_n(x)$	`eulerianpol`
Fibonacci number F_n	`fibonacci`(n)
Harmonic number $H_{n,r}=1^{-r}+\ldots+n^{-r}$	`harmonic`(n,r)
Stirling numbers $s(n,k)$ and $S(n,k)$	`stirling`$(n,k,\{flag\})$
number of partitions of n	`numbpart`(n)
k-th permutation on n letters	`numtoperm`(n,k)
... index k of permutation v	`permtonum`(v)
order of permutation p	`permorder`(p)
signature of permutation p	`permsign`(p)
cyclic decomposition of permutation p	`permcycles`(p)

Multiplicative groups $(\mathbf{Z}/N\mathbf{Z})^*$, $\mathbf{F}_q^*$

Euler ϕ-function	`eulerphi`(x)
multiplicative order of x (divides o)	`znorder`$(x,\{o\})$, `fforder`
primitive root mod q / `x.mod`	`znprimroot`(q), `ffprimroot`(x)
structure of $(\mathbf{Z}/n\mathbf{Z})^*$	`znstar`(n)
discrete logarithm of x in base g	`znlog`$(x,g,\{o\})$, `fflog`
Kronecker-Legendre symbol $(\frac{x}{y})$	`kronecker`(x,y)
quadratic Hilbert symbol (at p)	`hilbert`$(x,y,\{p\})$

Euclidean algorithm, continued fractions

CRT: solve $z\equiv x$ and $z\equiv y$	`chinese`(x,y)
minimal u,v so $xu+yv=\gcd(x,y)$	`gcdext`(x,y)
half-gcd algorithm	`halfgcd`(x,y)
continued fraction of x	`contfrac`$(x,\{b\},\{lmax\})$
last convergent of continued fraction x	`contfracpnqn`(x)
rational approximation to x (den. $\le B$)	`bestappr`$(x,\{B\})$
recognize $x\in\mathbf{C}$ as polmod mod $T\in\mathbf{Z}[X]$	`bestapprnf`(x,T)

Miscellaneous

integer square / n-th root of x	`sqrtint`(x), `sqrtnint`(x,n)
largest integer e s.t. $b^e\le b$, $e=\lfloor\log_b(x)\rfloor$	`logint`$(x,b,\{\&z\})$

Characters

Let $cyc=[d_1,\dots,d_k]$ represent an abelian group $G=\oplus(\mathbf{Z}/d_j\mathbf{Z})\cdot g_j$ or any structure G affording a `.cyc` method; e.g. `znstar`$(q,1)$ for Dirichlet characters. A character χ is coded by $[c_1,\dots,c_k]$ such that $\chi(g_j)=e(n_j/d_j)$.

$\chi\cdot\psi$; χ^{-1}; $\chi\cdot\psi^{-1}$; χ^k	`charmul`, `charconj`, `chardiv`, `charpow`
order of χ	`charorder`(cyc,χ)
kernel of χ	`charker`(cyc,χ)
$\chi(x)$, G a GP group structure	`chareval`$(G,\chi,x,\{z\})$
Galois orbits of characters	`chargalois`(G)

Dirichlet Characters

initialize $G=(\mathbf{Z}/q\mathbf{Z})^*$	`G = znstar`$(q,1)$
convert datum D to $[G,\chi]$	`znchar`(D)
is χ odd?	`zncharisodd`(G,χ)
real $\chi\to$ Kronecker symbol $(D/.)$	`znchartokronecker`(G,χ)
conductor of χ	`zncharconductor`(G,χ)
$[G_0,\chi_0]$ primitive attached to χ	`znchartoprimitive`(G,χ)
induce $\chi\in\hat{G}$ to $\mathbf{Z}/N\mathbf{Z}$	`zncharinduce`(G,χ,N)
χ_p	`znchardecompose`(G,χ,p)
$\prod_{p\mid(Q,N)}\chi_p$	`znchardecompose`(G,χ,Q)
complex Gauss sum $G_a(\chi)$	`znchargauss`(G,χ)

Conrey labelling

Conrey label $m\in(\mathbf{Z}/q\mathbf{Z})^*\to$ character	`znconreychar`(G,m)
character $\to$ Conrey label	`znconreyexp`(G,χ)
log on Conrey generators	`znconreylog`(G,m)
conductor of χ (χ_0 primitive)	`znconreyconductor`$(G,\chi,\{\chi_0\})$

True-False Tests

is x the disc. of a quadratic field?	`isfundamental`(x)
is x a prime?	`isprime`(x)
is x a strong pseudo-prime?	`ispseudoprime`(x)
is x square-free?	`issquarefree`(x)
is x a square?	`issquare`$(x,\{\&n\})$
is x a perfect power?	`ispower`$(x,\{k\},\{\&n\})$
is x a perfect power of a prime? ($x=p^n$)	`isprimepower`$(x,\&n\})$
... of a pseudoprime?	`ispseudoprimepower`$(x,\&n\})$
is x powerful?	`ispowerful`(x)
is x a totient? ($x=\varphi(n)$)	`istotient`$(x,\{\&n\})$
is x a polygonal number? ($x=P(s,n)$)	`ispolygonal`$(x,s,\{\&n\})$
is pol irreducible?	`polisirreducible`(pol)

Graphic Functions

crude graph of $expr$ between a and b	`plot`$(X=a,b,expr)$

High-resolution plot (immediate plot)

plot $expr$ between a and b	`ploth`$(X=a,b,expr,\{flag\},\{n\})$
plot points given by lists lx, ly	`plothraw`$(lx,ly,\{flag\})$
terminal dimensions	`plothsizes`$()$

Rectwindow functions

init window w, with size x,y	`plotinit`(w,x,y)
erase window w	`plotkill`(w)
copy w to w_2 with offset (dx,dy)	`plotcopy`(w,w_2,dx,dy)
clips contents of w	`plotclip`(w)
scale coordinates in w	`plotscale`(w,x_1,x_2,y_1,y_2)
`ploth` in w	`plotrecth`$(w,X=a,b,expr,\{flag\},\{n\})$
`plothraw` in w	`plotrecthraw`$(w,data,\{flag\})$
draw window w_1 at (x_1,y_1), ...	`plotdraw`$([[w_1,x_1,y_1],\dots])$

Low-level Rectwindow Functions

set current drawing color in w to c	`plotcolor`(w,c)
current position of cursor in w	`plotcursor`(w)
write s at cursor's position	`plotstring`(w,s)
move cursor to (x,y)	`plotmove`(w,x,y)
move cursor to $(x+dx,y+dy)$	`plotrmove`(w,dx,dy)
draw a box to (x_2,y_2)	`plotbox`(w,x_2,y_2)
draw a box to $(x+dx,y+dy)$	`plotrbox`(w,dx,dy)
draw polygon	`plotlines`$(w,lx,ly,\{flag\})$
draw points	`plotpoints`(w,lx,ly)
draw line to $(x+dx,y+dy)$	`plotrline`(w,dx,dy)
draw point $(x+dx,y+dy)$	`plotrpoint`(w,dx,dy)

Convert to Postscript or Scalable Vector Graphics

The format f is either `"ps"` or `"svg"`.

as `ploth`	`plothexport`$(f,X=a,b,expr,\{flag\},\{n\})$
as `plothraw`	`plothrawexport`$(f,lx,ly,\{flag\})$
as `plotdraw`	`plotexport`$(f,[[w_1,x_1,y_1],\dots])$

Based on an earlier version by Joseph H. Silverman

November 2022 v2.38. Copyright © 2022 K. Belabas
Permission is granted to make and distribute copies of this card provided the copyright and this permission notice are preserved on all copies.

Send comments and corrections to ⟨Karim.Belabas@math.u-bordeaux.fr⟩