

L-functions

(PARI-GP version 2.15.3)

Characters

A character on the abelian group $G = \sum_{j \leq k} (\mathbf{Z}/d_j \mathbf{Z}) \cdot g_j$, e.g. from **znstar**(**q**,1) $\leftrightarrow (\mathbf{Z}/q \mathbf{Z})^*$ or **bnrinit** $\leftrightarrow \text{Cl}_{\mathfrak{f}}(K)$, is coded by $\chi = [c_1, \dots, c_k]$ such that $\chi(g_j) = e(c_j/d_j)$. Our L -functions consider the attached *primitive* character.

Dirichlet characters $\chi_q(m, \cdot)$ in Conrey labelling system are alternatively concisely coded by **Mod**(**m**,**q**). Finally, a quadratic character ($D/\cdot$) can also be coded by the integer D .

L-function Constructors

An **Ldata** is a GP structure describing the functional equation for $L(s) = \sum_{n \geq 1} a_n n^{-s}$.

- Dirichlet coefficients given by closure $a : N \mapsto [a_1, \dots, a_N]$.
- Dirichlet coefficients $a^*(n)$ for dual L -function L^* .
- Euler factor $A = [a_1, \dots, a_d]$ for $\gamma_A(s) = \prod_i \Gamma_{\mathbf{R}}(s + a_i)$,
- classical weight k (values at s and $k - s$ are related),
- conductor N , $\Lambda(s) = N^{s/2} \gamma_A(s)$,
- root number ε ; $\Lambda(a, k - s) = \varepsilon \Lambda(a^*, s)$.
- polar part: list of $[\beta, P_{\beta}(x)]$.

An **Linit** is a GP structure containing an **Ldata** L and an evaluation *domain* fixing a maximal order of derivation m and bit accuracy (**realbitprecision**), together with complex ranges

- for L -function: $R = [c, w, h]$ (coding $|\Re z - c| \leq w$, $|\Im z| \leq h$); or $R = [w, h]$ (for $c = k/2$); or $R = [h]$ (for $c = k/2$, $w = 0$).
- for θ -function: $T = [\rho, \alpha]$ (for $|t| \geq \rho$, $|\arg t| \leq \alpha$); or $T = \rho$ (for $\alpha = 0$).

Ldata constructors

Riemann ζ	lfuncreate (1)
Dirichlet for quadratic char. ($D/\cdot$)	lfuncreate (D)
Dirichlet series $L(\chi_q(m, \cdot), s)$	lfuncreate (Mod (m , q))
Dedekind ζ_K , $K = \mathbf{Q}[x]/(T)$	lfuncreate (<i>bnf</i>), lfuncreate (T)
Hecke for $\chi \bmod \mathfrak{f}$	lfuncreate (<i>bnr</i> , χ)
Artin L -function	lfunartin (<i>nf</i> , <i>gal</i> , M , n)
Lattice Θ -function	lfunqf (Q)
From eigenform F	lfunmf (F)
Quotients of Dedekind $\eta: \prod_i \eta(m_{i,1} \cdot \tau)^{m_{i,2}}$	lfunetaquo (M)
$L(E, s)$, E elliptic curve	E = ellinit (...)
$L(\text{Sym}^m E, s)$, E elliptic curve	lfunsympow (E , m)
Genus 2 curve, $y^2 + xQ = P$	lfungenus2 ($[P, Q]$)
Hypergeometric motive $H(a, b; t)$	lfunhgm (hgmininit (a , b), t)

dual L function $\hat{L}$	lfundual (L)
$L_1 \cdot L_2$	lfunmul (L_1, L_2)
L_1/L_2	lfundiv (L_1, L_2)
$L(s - d)$	lfunshift (L, d)
$L(s) \cdot L(s - d)$	lfunshift ($L, d, 1$)
twist by Dirichlet character	lfuntwist (L, χ)

low-level constructor	lfuncreate ($[a, a^*, A, k, N, \textit{eps}, \textit{poles}]$)
check functional equation (at t)	lfuncheckfeq ($L, \{t\}$)
parameters $[N, k, A]$	lfunparams (L)

Linit constructors

initialize for L	lfuninit ($L, R, \{m = 0\}$)
initialize for θ	lfunthetainit ($L, \{T = 1\}, \{m = 0\}$)
cost of lfuninit	lfuncost ($L, R, \{m = 0\}$)
cost of lfunthetainit	lfunthetacost ($L, T, \{m = 0\}$)
Dedekind ζ_L , L abelian over a subfield	lfunabelianreinit

L-functions

L is an **Ldata** or an **Linit** (more efficient for many values).

Evaluate

$L^{(k)}(s)$	lfun ($L, s, \{k = 0\}$)
$\Lambda^{(k)}(s)$	lfunlambda ($L, s, \{k = 0\}$)
$\theta^{(k)}(t)$	lfuntheta ($L, t, \{k = 0\}$)

generalized Hardy Z -function at t **lfunhardy**(L, t)

Zeros

order of zero at $s = k/2$	lfunorderzero ($L, \{m = -1\}$)
zeros $s = k/2 + it$, $0 \leq t \leq T$	lfunzeros ($L, T, \{h\}$)

Dirichlet series and functional equation

$[a_n: 1 \leq n \leq N]$	lfunan (L, N)
Euler factor at p	lfuneuler (L, p)
conductor N of L	lfunconductor (L)
root number and residues	lfunrootres (L)

G-functions

Attached to inverse Mellin transform for $\gamma_A(s)$, $A = [a_1, \dots, a_d]$.
 initialize for G attached to A **gammamellinininit**(A)
 $G^{(k)}(t)$ **gammamellininiv**($G, t, \{k = 0\}$)
 asymp. expansion of $G^{(k)}(t)$ **gammamellininvasymp**($A, n, \{k = 0\}$)

Hypergeometric motives (HGM)

Hypergeometric templates

Below, H denotes an hypergeometric template from **hgminit**.
 HGM template from $A = (\alpha_j), B = (\beta_k)$ **hgminit**($A, \{B\}$)
 ...from cyclotomic parameters D, E **hgminit**($D, \{E\}$)
 ...from gamma vector **hgminit**(G)
 α and β parameters for H **hgmalph**(H)
 cyclotomic parameters (D, E) of H **hgmcyclo**(H)
 ...for all H of degree n **hgmbdegree**(n)
 gamma vector for H **hgmgamma**(H)
 twist A and B by $1/2$ **hgmtwist**(H)
 is H symmetrical at $t = 1$? **hgmissymmetrical**(H)
 parameters $[d, w, [P, T], M]$ for H **hgmparams**(H)

L-function

Let L be the L -function attached to the hypergeometric motive (H, t).

coefficient a_n of L	hgcoef (H, t, n)
coefficients $[a_1, \dots, a_n]$ of L	hgccoefs (H, t, n)
Euler factor at p	hgmeulerfactor (H, t, p)
...and valuation of local conductor	hgmeulerfactor ($H, t, p, \&e$)
return L as an Ldata	lfunhgm (H, t)

Based on an earlier version by Joseph H. Silverman

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