

# Pari-GP reference card

(PARI-GP version 2.15.3)

Note: optional arguments are surrounded by braces {}.

To start the calculator, type its name in the terminal: **gp**

To exit **gp**, type **quit**, **\q**, or **<C-D>** at prompt.

## Help

|                                        |                |
|----------------------------------------|----------------|
| describe function                      | ?function      |
| extended description                   | ??keyword      |
| list of relevant help topics           | ???pattern     |
| name of GP-1.39 function $f$ in GP-2.* | whatnow( $f$ ) |

## Input/Output

|                                        |                        |
|----------------------------------------|------------------------|
| previous result, the result before     | %, %`, %`` , etc.      |
| $n$ -th result since startup           | % $n$                  |
| separate multiple statements on line   | ;                      |
| extend statement on additional lines   | \                      |
| extend statements on several lines     | { $seq_1$ ; $seq_2$ ;} |
| comment                                | /* ... */              |
| one-line comment, rest of line ignored | \\ ...                 |

## Metacommands & Defaults

|                                          |                            |
|------------------------------------------|----------------------------|
| set default $d$ to $val$                 | default({ $d$ },{ $val$ }) |
| toggle timer on/off                      | #                          |
| print time for last result               | ##                         |
| print defaults                           | \d                         |
| set debug level to $n$                   | \g $n$                     |
| set memory debug level to $n$            | \gm $n$                    |
| set $n$ significant digits / bits        | \p $n$ , \pb $n$           |
| set $n$ terms in series                  | \ps $n$                    |
| quit GP                                  | \q                         |
| print the list of PARI types             | \t                         |
| print the list of user-defined functions | \u                         |
| read file into GP                        | \r $filename$              |
| set debuglevel for domain $D$ to $n$     | setdebug( $D,n$ )          |

## Debugger / break loop

|                                          |                           |
|------------------------------------------|---------------------------|
| get out of break loop                    | break or <C-D>            |
| go up/down $n$ frames                    | dbg_up({ $n$ }), dbg_down |
| set break point                          | breakpoint()              |
| examine object $o$                       | dbg_x( $o$ )              |
| current error data                       | dbg_err()                 |
| number of objects on heap and their size | getheap()                 |
| total size of objects on PARI stack      | getstack()                |

## PARI Types & Input Formats

|                                                     |                                       |
|-----------------------------------------------------|---------------------------------------|
| <b>t_INT</b> . Integers; hex, binary                | $\pm 31$ ; $\pm 0x1F$ , $\pm 0b101$   |
| <b>t_REAL</b> . Reals                               | $\pm 3.14$ , 6.022 E23                |
| <b>t_INTMOD</b> . Integers modulo $m$               | Mod( $n,m$ )                          |
| <b>t_FRAC</b> . Rational Numbers                    | $n/m$                                 |
| <b>t_FFELT</b> . Elt in finite field $\mathbf{F}_q$ | ffgen( $q$ , 't)                      |
| <b>t_COMPLEX</b> . Complex Numbers                  | $x + y * I$                           |
| <b>t_PADIC</b> . $p$ -adic Numbers                  | $x + O(p^k)$                          |
| <b>t_QUAD</b> . Quadratic Numbers                   | $x + y * \text{quadgen}(D, \{ 'w \})$ |
| <b>t_POLMOD</b> . Polynomials modulo $g$            | Mod( $f,g$ )                          |
| <b>t_POL</b> . Polynomials                          | $a * x^n + \dots + b$                 |
| <b>t_SER</b> . Power Series                         | $f + O(x^k)$                          |
| <b>t_RFRAC</b> . Rational Functions                 | $f/g$                                 |
| <b>t_QFB</b> . Binary quadratic form                | Qfb( $a,b,c$ )                        |
| <b>t_VEC/t_COL</b> . Row/Column Vectors             | [ $x,y,z$ ], [ $x,y,z$ ]~             |
| <b>t_VEC</b> integer range                          | [1..10]                               |

|                                          |                       |
|------------------------------------------|-----------------------|
| <b>t_VECSMALL</b> . Vector of small ints | Vecsmall([ $x,y,z$ ]) |
| <b>t_MAT</b> . Matrices                  | [ $a,b;c,d$ ]         |
| <b>t_LIST</b> . Lists                    | List([ $x,y,z$ ])     |
| <b>t_STR</b> . Strings                   | "abc"                 |
| <b>t_INFINITY</b> . $\pm\infty$          | +oo, -oo              |

## Reserved Variable Names

|                                                                                 |                       |
|---------------------------------------------------------------------------------|-----------------------|
| $\pi \approx 3.14$ , $\gamma \approx 0.57$ , $C \approx 0.91$ , $I = \sqrt{-1}$ | Pi, Euler, Catalan, I |
| Landau's big-oh notation                                                        | O                     |

## Information about an Object, Precision

|                                       |                                       |
|---------------------------------------|---------------------------------------|
| PARI type of object $x$               | type( $x$ )                           |
| length of $x$ / size of $x$ in memory | # $x$ , sizebyte( $x$ )               |
| real precision / bit precision of $x$ | precision( $x$ ), bitprecision( $x$ ) |
| $p$ -adic, series prec. of $x$        | padicprec( $x,p$ ), serprec( $x,v$ )  |
| current dynamic precision             | getlocalprec, getlocalbitprec         |

## Operators

|                                                                    |                                                                  |
|--------------------------------------------------------------------|------------------------------------------------------------------|
| basic operations                                                   | +, -, *, /, ^, sqr                                               |
| $i \leftarrow i+1$ , $i \leftarrow i-1$ , $i \leftarrow i*j$ , ... | i++, i--, i*=j,...                                               |
| Euclidean quotient, remainder                                      | $x \backslash y$ , $x \backslash y$ , $x \% y$ , divrem( $x,y$ ) |
| shift $x$ left or right $n$ bits                                   | $x << n$ , $x >> n$ or shift( $x, \pm n$ )                       |
| multiply by $2^n$                                                  | shiftmul( $x,n$ )                                                |
| comparison operators                                               | <=, <, >=, >, ==, !=, ==, lex, cmp                               |
| boolean operators (or, and, not)                                   | , &&, !                                                          |
| bit operations                                                     | bitand, bitneg, bitor, bitxor, bitnegimply                       |
| maximum/minimum of $x$ and $y$                                     | max( $x,y$ ), min( $x,y$ )                                       |
| sign of $x$ (gives $-1, 0, 1$ )                                    | sign( $x$ )                                                      |
| binary exponent of $x$                                             | exponent( $x$ )                                                  |
| derivative of $f$ , 2nd derivative, etc.                           | $f'$ , $f''$ , ...                                               |
| differential operator                                              | diffop( $f,v,d,\{n=1\}$ )                                        |
| quote operator (formal variable)                                   | 'x                                                               |
| assignment                                                         | x = value                                                        |
| simultaneous assignment $x \leftarrow v[1]$ , $y \leftarrow v[2]$  | [x,y] = v                                                        |

## Select Components

|                                                     |                                      |
|-----------------------------------------------------|--------------------------------------|
| <i>Caveat</i> : components start at index $n = 1$ . |                                      |
| $n$ -th component of $x$                            | component( $x,n$ )                   |
| $n$ -th component of vector/list $x$                | $x[n]$                               |
| components $a, a+1, \dots, b$ of vector $x$         | $x[a..b]$                            |
| $(m,n)$ -th component of matrix $x$                 | $x[m,n]$                             |
| row $m$ or column $n$ of matrix $x$                 | $x[m,]$ , $x[,n]$                    |
| numerator/denominator of $x$                        | numerator( $x$ ), denominator( $x$ ) |

## Random Numbers

|                                 |                                   |
|---------------------------------|-----------------------------------|
| random integer/prime in $[0,N[$ | random( $N$ ), randomprime( $N$ ) |
| get/set random seed             | getrand, setrand( $s$ )           |

## Conversions

|                                                  |                        |
|--------------------------------------------------|------------------------|
| to vector, matrix, vec. of small ints            | Col/Vec, Mat, Vecsmall |
| to list, set, map, string                        | List, Set, Map, Str    |
| create $(x \bmod y)$                             | Mod( $x,y$ )           |
| make $x$ a polynomial of $v$                     | Pol( $x,\{v\}$ )       |
| variants of Pol <i>et al.</i> , in reverse order | Polrev, Vecrev, Colrev |
| make $x$ a power series of $v$                   | Ser( $x,\{v\}$ )       |
| convert $x$ to simplest possible type            | simplify( $x$ )        |
| object $x$ with real precision $n$               | precision( $x,n$ )     |
| object $x$ with bit precision $n$                | bitprecision( $x,n$ )  |
| set precision to $p$ digits in dynamic scope     | localprec( $p$ )       |
| set precision to $p$ bits in dynamic scope       | localbitprec( $p$ )    |

## Character strings

|                                                                         |                         |
|-------------------------------------------------------------------------|-------------------------|
| convert to TeX representation                                           | strtex( $x$ )           |
| string from bytes / from format+args                                    | strchr, sprintf         |
| split string / join strings                                             | strsplit, strjoin       |
| convert time $t$ ms. to h, m, s, ms format                              | strtime( $t$ )          |
| <b>Conjugates and Lifts</b>                                             |                         |
| conjugate of a number $x$                                               | conj( $x$ )             |
| norm of $x$ , product with conjugate                                    | norm( $x$ )             |
| $L^p$ norm of $x$ ( $L^\infty$ if no $p$ )                              | normlp( $x,\{p\}$ )     |
| square of $L^2$ norm of $x$                                             | norml2( $x$ )           |
| lift of $x$ from Mods and $p$ -adics                                    | lift, centerlift( $x$ ) |
| recursive lift                                                          | liftall                 |
| lift all <b>t_INT</b> and <b>t_PADIC</b> ( $\rightarrow$ <b>t_INT</b> ) | liftint                 |
| lift all <b>t_POLMOD</b> ( $\rightarrow$ <b>t_POL</b> )                 | liftpol                 |

## Lists, Sets & Maps

|                                                                        |                             |
|------------------------------------------------------------------------|-----------------------------|
| <b>Sets</b> (= row vector with strictly increasing entries w.r.t. cmp) |                             |
| intersection of sets $x$ and $y$                                       | setintersect( $x,y$ )       |
| set of elements in $x$ not belonging to $y$                            | setminus( $x,y$ )           |
| symmetric difference $x \Delta y$                                      | setdelta( $x,y$ )           |
| union of sets $x$ and $y$                                              | setunion( $x,y$ )           |
| does $y$ belong to the set $x$                                         | setsearch( $x,y,\{flag\}$ ) |
| set of all $f(x,y)$ , $x \in X$ , $y \in Y$                            | setbinop( $f,X,Y$ )         |
| is $x$ a set ?                                                         | setisset( $x$ )             |

|                                                           |                               |
|-----------------------------------------------------------|-------------------------------|
| <b>Lists</b> . create empty list: $L = \text{List}()$     |                               |
| append $x$ to list $L$                                    | listput( $L,x,\{i\}$ )        |
| remove $i$ -th component from list $L$                    | listpop( $L,\{i\}$ )          |
| insert $x$ in list $L$ at position $i$                    | listinsert( $L,x,i$ )         |
| sort the list $L$ in place                                | listsort( $L,\{flag\}$ )      |
| <b>Maps</b> . create empty dictionary: $M = \text{Map}()$ |                               |
| attach value $v$ to key $k$                               | mapput( $M,k,v$ )             |
| recover value attach to key $k$ or error                  | mapget( $M,k$ )               |
| is key $k$ in the dict? (set $v$ to $M(k)$ )              | mapisdefined( $M,k,\{\&v\}$ ) |
| remove $k$ from map domain                                | mapdelete( $M,k$ )            |

## GP Programming

|                                                                          |                                 |
|--------------------------------------------------------------------------|---------------------------------|
| <b>User functions and closures</b>                                       |                                 |
| $x,y$ are formal parameters; $y$ defaults to Pi if parameter omitted;    |                                 |
| $z,t$ are local variables (lexical scope), $z$ initialized to 1.         |                                 |
| fun(x, y=Pi) = my(z=1, t); seq                                           |                                 |
| fun = (x, y=Pi) -> my(z=1, t); seq                                       |                                 |
| attach help message $h$ to $s$                                           | addhelp( $s,h$ )                |
| undefine symbol $s$ (also kills help)                                    | kill(s)                         |
| <b>Control Statements</b> ( $X$ : formal parameter in expression $seq$ ) |                                 |
| if $a \neq 0$ , evaluate $seq_1$ , else $seq_2$                          | if( $a,\{seq_1\},\{seq_2\}$ )   |
| eval. $seq$ for $a \leq X \leq b$                                        | for( $X = a,b,seq$ )            |
| ...for $X \in v$                                                         | foreach( $v,X,seq$ )            |
| ...for primes $a \leq X \leq b$                                          | forprime( $X = a,b,seq$ )       |
| ...for primes $\equiv a \pmod q$                                         | forprimestep( $X = a,b,q,seq$ ) |
| ...for composites $a \leq X \leq b$                                      | forcomposite( $X = a,b,seq$ )   |
| ...for $a \leq X \leq b$ stepping $s$                                    | forstep( $X = a,b,s,seq$ )      |
| ...for $X$ dividing $n$                                                  | fordiv( $n,X,seq$ )             |
| ... $X = [n, factor(n)]$ , $a \leq n \leq b$                             | forfactored( $X = a,b,seq$ )    |
| ...as above, $n$ squarefree                                              | forsquarefree( $X = a,b,seq$ )  |
| ... $X = [d, factor(d)]$ , $d \mid n$                                    | fordivfactored( $n,X,seq$ )     |
| multivariable for, lex ordering                                          | forvec( $X = v,seq$ )           |

```

loop over partitions of  $n$ 
... permutations of  $S$ 
... subsets of  $\{1, \dots, n\}$ 
...  $k$ -subsets of  $\{1, \dots, n\}$ 
... vectors  $v, q(v) \leq B$ ;  $q > 0$ 
...  $H < G$  finite abelian group
evaluate  $seq$  until  $a \neq 0$ 
while  $a \neq 0$ , evaluate  $seq$ 
exit  $n$  innermost enclosing loops
start new iteration of  $n$ -th enclosing loop
return  $x$  from current subroutine
Exceptions, warnings
raise an exception / warning
type of error message  $E$ 
try  $seq_1$ , evaluate  $seq_2$  on error
Functions with closure arguments / results
number of arguments of  $f$ 
select from  $v$  according to  $f$ 
apply  $f$  to all entries in  $v$ 
evaluate  $f(a_1, \dots, a_n)$ 
evaluate  $f(\dots f(f(a_1, a_2), a_3) \dots, a_n)$ 
calling function as closure
Sums & Products
sum  $X = a$  to  $X = b$ , initialized at  $x$ 
sum entries of vector  $v$ 
product of all vector entries
sum  $expr$  over divisors of  $n$ 
... assuming  $expr$  multiplicative
product  $a \leq X \leq b$ , initialized at  $x$ 
product over primes  $a \leq X \leq b$ 
Sorting
sort  $x$  by  $k$ -th component
min.  $m$  of  $x$  ( $m = x[i]$ ), max.
does  $y$  belong to  $x$ , sorted wrt.  $f$ 
 $\prod g^x \rightarrow$  factorization ( $\Rightarrow$  sorted, unique  $g$ )
Input/Output
print with/without  $\backslash n$ , TeX format
pretty print matrix
print fields with separator
formatted printing
write  $args$  to file
write  $x$  in binary format
read file into GP
... return as vector of lines
... return as vector of strings
read a string from keyboard
Files and file descriptors
File descriptors allow efficient small consecutive reads or writes
from or to a given file. The argument  $n$  below is always a descriptor,
attached to a file in r(ead), w(rite) or a(ppend) mode.
get descriptor  $n$  for file  $path$  in given  $mode$ 
... from shell  $cmd$  output (pipe)
close descriptor
commit pending write operations
read logical line from file
... raw line from file
write  $s \backslash n$  to file
... write  $s$  to file

```

```

forpart( $p = n, seq$ )
forperm( $S, p, seq$ )
forsubset( $n, p, seq$ )
forsubset( $[n, k], p, seq$ )
forqfvec( $v, q, b, seq$ )
forsubgroup( $H = G$ )
until( $a, seq$ )
while( $a, seq$ )
break( $\{n\}$ )
next( $\{n\}$ )
return( $\{x\}$ )

error(), warning()
errname( $E$ )
iferr( $seq_1, E, seq_2$ )

Results
arity( $f$ )
select( $f, v$ )
apply( $f, v$ )
call( $f, a$ )
fold( $f, a$ )
self()

sum( $X = a, b, expr, \{x\}$ )
vecsum( $v$ )
vecprod( $v$ )
sumdiv( $n, X, expr$ )
sumdivmult( $n, X, expr$ )
prod( $X = a, b, expr, \{x\}$ )
prodeuler( $X = a, b, expr$ )

vecsort( $x, \{k\}, \{fl = 0\}$ )
vecmin( $x, \{\&i\}$ ), vecmax
vecsearch( $x, y, \{f\}$ )
matreduce( $m$ )

print, print1, printtex
printp
printsep( $sep, \dots$ ), printsep1
printf()

write, write1, writetex( $file, args$ )
writebin( $file, x$ )
read( $\{file\}$ )
readvec( $\{file\}$ )
readstr( $\{file\}$ )
input()

fileclose( $n$ )
fileflush( $n$ )
fileread( $n$ )
filereadstr( $n$ )
filewrite( $n, s$ )
filewrite1( $n, s$ )

```

# Pari-GP reference card

(PARI-GP version 2.15.3)

## Timers

CPU time in  $ms$  and reset timer  
CPU time in  $ms$  since gp startup  
time in  $ms$  since UNIX Epoch  
timeout command after  $s$  seconds

## Interface with system

allocates a new stack of  $s$  bytes  
alias *old* to *new*  
install function from library  
execute system command  $a$   
... and feed result to GP  
... returning GP string  
get \$VAR from environment  
expand env. variable in string

```

gettime()
getabstime()
getwalltime()
alarm( $s, expr$ )

allocatemem( $\{s\}$ )
alias( $new, old$ )
install( $f, code, \{gpf\}, \{lib\}$ )
system( $a$ )
extern( $a$ )
externstr( $a$ )
getenv("VAR")
strexpand( $x$ )

```

## Parallel evaluation

These functions evaluate their arguments in parallel (pthreads or MPI); args. must not access global variables (use **export** for this) and must be free of side effects. Enabled if threading engine is not *single* in gp header.

```

evaluate  $f$  on  $x[1], \dots, x[n]$ 
evaluate closures  $f[1], \dots, f[n]$ 
as select
as sum
as vector
eval  $f$  for  $i = a, \dots, b$ 
... for each element  $x$  in  $v$ 
... for  $p$  prime in  $[a, b]$ 
... for  $p = a \bmod q$ 
... multivariate
parforvec( $X = v, f, \{r\}, \{f_2\}, \{flag\}$ )
export  $x$  to parallel world
... all dynamic variables
frees exported value  $x$ 
... all exported values

```

```

parapply( $f, x$ )
pareval( $f$ )
parselect( $f, A, \{flag\}$ )
parsum( $i = a, b, expr$ )
parvector( $n, i, \{expr\}$ )
parfor( $i = a, \{b\}, f, \{r\}, \{f_2\}$ )
parforeach( $v, x, f, \{r\}, \{f_2\}$ )
parforprime( $p = a, \{b\}, f, \{r\}, \{f_2\}$ )
parforprimestep( $p = a, \{b\}, q, f, \{r\}, \{f_2\}$ )

```

## Linear Algebra

dimensions of matrix  $x$   
multiply two matrices  
... assuming result is diagonal  
concatenation of  $x$  and  $y$   
extract components of  $x$   
transpose of vector or matrix  $x$   
adjoint of the matrix  $x$   
eigenvectors/values of matrix  $x$   
characteristic/minimal polynomial of  $x$   
trace/determinant of matrix  $x$   
permanent of matrix  $x$   
Frobenius form of  $x$   
QR decomposition  
apply **matqr**'s transform to  $v$

```

matsize( $x$ )
 $x * y$ 
matmultodiagonal( $x, y$ )
concat( $x, \{y\}$ )
vecextract( $x, y, \{z\}$ )
 $x \sim$ , mattranspose( $x$ )
matadjoint( $x$ )
mateigen( $x$ )
charpoly( $x$ ), minpoly( $x$ )
trace( $x$ ), matdet( $x$ )
matpermanent( $x$ )
matfrobenius( $x$ )
matqr( $x$ )
mathouseholder( $Q, v$ )

```

## Constructors & Special Matrices

```

{ $g(x): x \in v$  s.t.  $f(x)$ }
{ $x: x \in v$  s.t.  $f(x)$ }
{ $g(x): x \in v$ }
row vec. of  $expr$  eval'ed at  $1 \leq i \leq n$ 
col. vec. of  $expr$  eval'ed at  $1 \leq i \leq n$ 
vector of small ints

```

```

[g(x) | x <- v, f(x)]
[x | x <- v, f(x)]
[g(x) | x <- v]
vector( $n, \{i\}, \{expr\}$ )
vectorv( $n, \{i\}, \{expr\}$ )
vectorsmall( $n, \{i\}, \{expr\}$ )

```

```

[ $c, c \cdot x, \dots, c \cdot x^n$ ]
[ $1, 2^x, \dots, n^x$ ]
matrix  $1 \leq i \leq m, 1 \leq j \leq n$ 
define matrix by blocks
diagonal matrix with diagonal  $x$ 
is  $x$  diagonal?
 $x \cdot \text{matdiagonal}(d)$ 
 $n \times n$  identity matrix
Hessenberg form of square matrix  $x$ 
 $n \times n$  Hilbert matrix  $H_{ij} = (i + j - 1)^{-1}$ 
 $n \times n$  Pascal triangle
companion matrix to polynomial  $x$ 
Sylvester matrix of  $x$  and  $y$ 

```

## Gaussian elimination

```

kernel of matrix  $x$ 
intersection of column spaces of  $x$  and  $y$ 
solve  $MX = B$  ( $M$  invertible)
one sol of  $M * X = B$ 
basis for image of matrix  $x$ 
columns of  $x$  not in matimage
supplement columns of  $x$  to get basis
rows, cols to extract invertible matrix
rank of the matrix  $x$ 
solve  $MX = B \bmod D$ 
image mod  $D$ 
kernel mod  $D$ 
inverse mod  $D$ 
determinant mod  $D$ 

```

## Lattices & Quadratic Forms

### Quadratic forms

```

evaluate  ${}^t x Q y$ 
evaluate  ${}^t x Q x$ 
signature of quad form  ${}^t y * x * y$ 
decomp into squares of  ${}^t y * x * y$ 
eigenvalues/vectors for real symmetric  $x$ 

```

### HNF and SNF

```

upper triangular Hermite Normal Form
HNF of  $x$  where  $d$  is a multiple of  $\det(x)$ 
multiple of  $\det(x)$ 
HNF of  $(x \mid \text{diagonal}(D))$ 
elementary divisors of  $x$ 
 $q$ -rank from elementary divisors
elementary divisors of  $\mathbf{Z}[a]/(f'(a))$ 
integer kernel of  $x$ 
 $\mathbf{Z}$ -module  $\leftrightarrow$   $\mathbf{Q}$ -vector space

```

### Lattices

```

LLL-algorithm applied to columns of  $x$ 
... for Gram matrix of lattice
find up to  $m$  sols of qfnorm( $x, y$ )  $\leq b$ 
 $v, v[i] :=$  number of  $y$  s.t. qfnorm( $x, y$ ) =  $i$ 
perfection rank of  $x$ 
find isomorphism between  $q$  and  $Q$ 
precompute for isomorphism test with  $q$ 
automorphism group of  $q$ 

```

Based on an earlier version by Joseph H. Silverman

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Send comments and corrections to (Karim.Belabas@math.u-bordeaux.fr)

convert `qfauto` for GAP/Magma `qfautoexport( $G, \{flag\}$ )`  
orbits of  $V$  under  $G \subset \mathrm{GL}(V)$  `qforbits( $G, V$ )`

Polynomials & Rational Functions

all defined polynomial variables `variables()`  
get var. of highest priority (higher than  $v$ ) `varhigher( $name, \{v\}$ )`  
... of lowest priority (lower than  $v$ ) `varlower( $name, \{v\}$ )`

Coefficients, variables and basic operators

degree of  $f$  `poldegree( $f$ )`  
coef. of degree  $n$  of  $f$ , leading coef. `polcoef( $f, n$ ), pollead`  
main variable / all variables in  $f$  `variable( $f$ ), variables( $f$ )`  
replace  $x$  by  $y$  in  $f$  `subst( $f, x, y$ )`  
evaluate  $f$  replacing vars by their value `eval( $f$ )`  
replace polynomial expr.  $T(x)$  by  $y$  in  $f$  `substpol( $f, T, y$ )`  
replace  $x_1, \dots, x_n$  by  $y_1, \dots, y_n$  in  $f$  `substvec( $f, x, y$ )`  
 $f \in A[x]$ ; reciprocal polynomial  $x^{\deg f} f\left(\frac{1}{x}\right)$  `polrecip( $f$ )`  
gcd of coefficients of  $f$  `content( $f$ )`  
derivative of  $f$  w.r.t.  $x$  `deriv( $f, \{x\}$ )`  
...  $n$ -th derivative of  $f$  `derivn( $f, n, \{x\}$ )`  
formal integral of  $f$  w.r.t.  $x$  `intformal( $f, \{x\}$ )`  
formal sum of  $f$  w.r.t.  $x$  `sumformal( $f, \{x\}$ )`

Constructors & Special Polynomials

interpolation polynomial at  $(x[1], y[1]), \dots, (x[n], y[n])$ , evaluated at  $t$ , with error estimate  $e$  `polinterpolate( $x, \{y\}, \{t\}, \{&e\}$ )`  
 $T_n/U_n, H_n$  `polchebyshev( $n$ ), polhermite( $n$ )`  
 $P_n, L_n^{(\alpha)}$  `pollegendre( $n$ ), pollaguerre( $n, a$ )`  
 $n$ -th cyclotomic polynomial  $\Phi_n$  `polcyclo( $n$ )`  
return  $n$  if  $f = \Phi_n$ , else 0 `poliscyclo( $f$ )`  
is  $f$  a product of cyclotomic polynomials? `poliscycloprod( $f$ )`  
Zagier's polynomial of index  $(n, m)$  `polzagier( $n, m$ )`

Resultant, elimination

discriminant of polynomial  $f$  `poldisc( $f$ )`  
find factors of `poldisc( $f$ )` `poldiscfactors( $f$ )`  
resultant  $R = \mathrm{Res}_v(f, g)$  `polresultant( $f, g, \{v\}$ )`  
 $[u, v, R], xu + yv = \mathrm{Res}_v(f, g)$  `polresultantext( $x, y, \{v\}$ )`  
solve Thue equation  $f(x, y) = a$  `thue( $t, a, \{sol\}$ )`  
initialize  $t$  for Thue equation solver `thueinit( $f$ )`

Roots and Factorization (Complex/Real)

complex roots of  $f$  `polroots( $f$ )`  
bound complex roots of  $f$  `polrootsbound( $f$ )`  
number of real roots of  $f$  (in  $[a, b]$ ) `polsturm( $f, \{[a, b]\}$ )`  
real roots of  $f$  (in  $[a, b]$ ) `polrootsreal( $f, \{[a, b]\}$ )`  
complex embeddings of `t_POLMOD`  $z$  `conjvec( $z$ )`

Roots and Factorization (Finite fields)

factor  $f$  mod  $p$ , roots `factormod( $f, p$ ), polrootsmod`  
factor  $f$  over  $\mathbf{F}_p[x]/(T)$ , roots `factormod( $f, [T, p]$ ), polrootsmod`  
squarefree factorization of  $f$  in  $\mathbf{F}_q[x]$  `factormodSQF( $f, \{D\}$ )`  
distinct degree factorization of  $f$  in  $\mathbf{F}_q[x]$  `factormodDDF( $f, \{D\}$ )`  
factor  $n$ -th cyclotomic pol.  $\Phi_n$  mod  $p$  `factormodcyclo( $n, p$ )`

Roots and Factorization ( $p$ -adic fields)

factor  $f$  over  $\mathbf{Q}_p$ , roots `factorpadic( $f, p, r$ ), polrootspadic`  
 $p$ -adic root of  $f$  congruent to  $a$  mod  $p$  `padicappr( $f, a$ )`  
Newton polygon of  $f$  for prime  $p$  `newtonpoly( $f, p$ )`  
Hensel lift  $A/\mathrm{lc}(A) = \prod_i B[i]$  mod  $p^e$  `polhensellift( $A, B, p, e$ )`  
 $T = \prod (x - z_i) \mapsto \prod [x - \omega(z_i)] \in \mathbf{Z}_p[x]$  `polteichmuller( $T, p, e$ )`  
extensions of  $\mathbf{Q}_p$  of degree  $N$  `padicfields( $p, N$ )`

Pari-GP reference card

(PARI-GP version 2.15.3)

Roots and Factorization (Miscellaneous)

symmetric powers of roots of  $f$  up to  $n$  `polsym( $f, n$ )`  
Graeffe transform of  $f, g(x^2) = f(x)f(-x)$  `polgraeffe( $f$ )`  
factor  $f$  over coefficient field `factor( $f$ )`  
cyclotomic factors of  $f \in \mathbf{Q}[X]$  `polcyclofactors( $f$ )`

Finite Fields

A finite field is encoded by any element (`t_FFELT`).  
find irreducible  $T \in \mathbf{F}_p[x]$ ,  $\deg T = n$  `ffinit( $p, n, \{x\}$ )`  
Create  $t$  in  $\mathbf{F}_q \simeq \mathbf{F}_p[t]/(T)$  `t = ffgen( $T, 't$ )`  
... indirectly, with implicit  $T$  `t = ffgen( $q, 't$ ); T = t.mod`  
map  $m$  from  $\mathbf{F}_q \ni a$  to  $\mathbf{F}_{q^k} \ni b$  `m = ffbend( $a, b$ )`  
build  $K = \mathbf{F}_q[x]/(P)$  extending  $\mathbf{F}_q \ni a$ , `ffextend( $a, P$ )`  
evaluate map  $m$  on  $x$  `ffmap( $m, x$ )`  
inverse map of  $m$  `ffinvmap( $m$ )`  
compose maps  $m \circ n$  `ffcompomap( $m, n$ )`  
 $x$  as polmod over codomain of map  $m$  `ffmaprel( $m, x$ )`  
 $F^n$  over  $\mathbf{F}_q \ni a$  `fffrobenius( $a, n$ )`  
 $\#\{\text{monic irred. } T \in \mathbf{F}_q[x], \deg T = n\}$  `ffnbirred( $q, n$ )`

Formal &  $p$ -adic Series

truncate power series or  $p$ -adic number `truncate( $x$ )`  
valuation of  $x$  at  $p$  `valuation( $x, p$ )`  
**Dirichlet and Power Series**  
Taylor expansion around 0 of  $f$  w.r.t.  $x$  `taylor( $f, x$ )`  
Laurent series of closure  $F$  up to  $x^k$  `laurentseries( $f, k$ )`  
 $\sum a_k b_k t^k$  from  $\sum a_k t^k$  and  $\sum b_k t^k$  `serconvol( $a, b$ )`  
 $f = \sum a_k t^k$  from  $\sum (a_k/k!) t^k$  `serlaplace( $f$ )`  
reverse power series  $F$  so  $F(f(x)) = x$  `serreverse( $f$ )`  
remove terms of degree  $< n$  in  $f$  `serchop( $f, n$ )`  
Dirichlet series multiplication / division `dirmul, dirdiv( $x, y$ )`  
Dirichlet Euler product ( $b$  terms) `direuler( $p = a, b, expr$ )`

Transcendental and  $p$ -adic Functions

real, imaginary part of  $x$  `real( $x$ ), imag( $x$ )`  
absolute value, argument of  $x$  `abs( $x$ ), arg( $x$ )`  
square/ $n$ th root of  $x$  `sqrt( $x$ ), sqrtsn( $x, n, \{&z\}$ )`  
all  $n$ -th roots of 1 `rootsof1( $n$ )`  
FFT of  $[f_0, \dots, f_{n-1}]$  `w = fftinit( $n$ ), fft/fftinw( $w, f$ )`  
trig functions `sin, cos, tan, cotan, sinc`  
inverse trig functions `asin, acos, atan`  
hyperbolic functions `sinh, cosh, tanh, cotanh`  
inverse hyperbolic functions `asinh, acosh, atanh`  
 $\log(x), \log(1+x), e^x, e^x - 1$  `log, loglp, exp, expm1`  
Euler  $\Gamma$  function,  $\log \Gamma, \Gamma'/\Gamma$  `gamma, lngamma, psi`  
half-integer gamma function  $\Gamma(n+1/2)$  `gammah( $n$ )`  
Riemann's zeta  $\zeta(s) = \sum n^{-s}$  `zeta( $s$ )`  
 $\sum_{1 \leq n \leq N} n^s$  `dirpowerssum( $N, s$ )`  
Hurwitz's  $\zeta(s, x) = \sum (n+x)^{-s}$  `zetahurwitz( $s, x$ )`  
Lerch  $\Phi(z, s, x) = \sum z^n (n+x)^{-s}$  `lerchphi( $z, s, x$ )`  
Lerch  $L(s, x, t) = \Phi(e^{2i\pi t}, s, x)$  `lerchzeta( $s, x, t$ )`  
multiple zeta value (MZV),  $\zeta(s_1, \dots, s_k)$  `zetamult( $s, \{T\}$ )`  
all MZVs for weight  $\sum s_i = n$  `zetamultall( $n$ )`  
convert MZV id to  $[s_1, \dots, s_k]$  `zetamultconvert( $f, \{flag\}$ )`  
MZV dual sequence `zetamultdual( $s$ )`  
multiple polylog  $Li_{s_1, \dots, s_k}(z_1, \dots, z_k)$  `polylogmult( $s, z$ )`

incomplete  $\Gamma$  function ( $y = \Gamma(s)$ ) `incgam( $s, x, \{y\}$ )`  
complementary incomplete  $\Gamma$  `incgamc( $s, x$ )`  
 $\int_x^\infty e^{-t} dt/t, (2/\sqrt{\pi}) \int_x^\infty e^{-t^2} dt$  `eint1, erfc`  
elliptic integral of 1st and 2nd kind `ellK( $k$ ), ellE( $k$ )`  
dilogarithm of  $x$  `dilog( $x$ )`  
 $m$ -th polylogarithm of  $x$  `polylog( $m, x, \{flag\}$ )`  
 $U$ -confluent hypergeometric function `hyperu( $a, b, u$ )`  
Hypergeometric  ${}_pF_q(A, B; z)$  `hypergeom( $A, B, z$ )`  
Bessel  $J_n(x), J_{n+1/2}(x)$  `besselj( $n, x$ ), besseljh( $n, x$ )`  
Bessel  $I_\nu, K_\nu, H_\nu^1, H_\nu^2, Y_\nu$  `(bessel)i, k, h1, h2, y`  
 $k$ -th zero of  $J_\nu(x)$  `besseljzero( $nu, \{k = 1\}$ )`  
 $k$ -th zero of  $Y_\nu(x)$  `besselyzero( $nu, \{k = 1\}$ )`  
Airy functions  $A_i(x), B_i(x)$  `airy( $x$ )`  
Lambert  $W: x$  s.t.  $xe^x = y$  `lambertw( $y$ )`  
Teichmuller character of  $p$ -adic  $x$  `teichmuller( $x$ )`

Iterations, Sums & Products

Numerical integration for meromorphic functions

Behaviour at endpoint for Double Exponential (DE) methods: either a scalar ( $a \in \mathbf{C}$ , regular) or  $\pm\infty$  (decreasing at least as  $x^{-2}$ ) or  
 $(x-a)^{-\alpha}$  singularity `[ $a, \alpha$ ]`  
exponential decrease  $e^{-\alpha|x|}$  `[ $\pm\infty, \alpha$ ],  $\alpha > 0$`   
slow decrease  $|x|^\alpha$  `...  $\alpha < -1$`   
oscillating as  $\cos(kx)$   `$\alpha = k\mathbf{I}, k > 0$`   
oscillating as  $\sin(kx)$   `$\alpha = -k\mathbf{I}, k > 0$`

numerical integration `intnum( $x = a, b, f, \{T\}$ )`  
weights  $T$  for intnum `intnuminit( $a, b, \{m\}$ )`  
weights  $T$  incl. kernel  $K$  `intfuncinit( $t = a, b, K, \{m\}$ )`  
integrate  $(2i\pi)^{-1} f$  on circle  $|z-a| = R$  `intcirc( $x = a, R, f, \{T\}$ )`  
**Other integration methods**  
 $n$ -point Gauss-Legendre `intnumgauss( $x = a, b, f, \{n\}$ )`  
weights for  $n$ -point Gauss-Legendre `intnumgaussinit( $\{n\}$ )`  
quasi-periodic function, period  $2H$  `intnumosc( $x = a, f, H$ )`  
Romberg (low accuracy) `intnumromb( $x = a, b, f, \{flag\}$ )`

Numerical summation

sum of series  $f(n), n \geq a$  (low accuracy) `suminf( $n = a, expr$ )`  
sum of alternating/positive series `sumalt, sumpos`  
sum of series using Euler-Maclaurin `sumnum( $n = a, f, \{T\}$ )`  
... Sidi summation `sumnumsidi( $n = a, f$ )`  
 $\sum_{n \geq a} F(n)$ ,  $F$  rational function `sumnumrat( $F, a$ )`  
...  $\sum_{p \geq a} F(p^s)$  `sumeulerrat( $F, \{s = 1\}, \{a = 2\}$ )`  
weights for sumnum,  $a$  as in DE `sumnuminit( $\{\infty, a\}$ )`  
sum of series by Monien summation `sumnummonien( $n = a, f, \{T\}$ )`  
weights for sumnummonien `sumnummonieninit( $\{\infty, a\}$ )`  
sum of series using Abel-Plana `sumnumap( $n = a, f, \{T\}$ )`  
weights for sumnumap,  $a$  as in DE `sumnumapinit( $\{\infty, a\}$ )`  
sum of series using Lagrange `sumnumlagrange( $n = a, f, \{T\}$ )`  
weights for sumnumlagrange `sumnumlagrangeinit`

Products

product  $a \leq X \leq b$ , initialized at  $x$  `prod( $X = a, b, expr, \{x\}$ )`  
product over primes  $a \leq X \leq b$  `prodeuler( $X = a, b, expr$ )`  
infinite product  $a \leq X \leq \infty$  `prodinf( $X = a, expr$ )`  
 $\prod_{n \geq a} F(n)$ ,  $F$  rational function `prodnumrat( $F, a$ )`  
 $\prod_{p \geq a} F(p^s)$  `prodeulerrat( $F, \{s = 1\}, \{a = 2\}$ )`

Other numerical methods

|                                               |                                                                   |
|-----------------------------------------------|-------------------------------------------------------------------|
| real root of $f$ in $[a, b]$ ; bracketed root | <code>solve(<math>X = a, b, f</math>)</code>                      |
| ...interval splitting, step $s$               | <code>solvestep(<math>X = a, b, s, f, \{flag = 0\}</math>)</code> |
| limit of $f(t)$ , $t \rightarrow \infty$      | <code>limitnum(<math>f, \{\alpha\}</math>)</code>                 |
| asymptotic expansion of $f$ (rational)        | <code>asypnum(<math>f, \{\alpha\}</math>)</code>                  |
| ... $N + 1$ terms as floats                   | <code>asypnumraw(<math>f, N, \{\alpha\}</math>)</code>            |
| numerical derivation w.r.t $x$ : $f'(a)$      | <code>derivnum(<math>x = a, f</math>)</code>                      |
| evaluate continued fraction $F$ at $t$        | <code>contfraceval(<math>F, t, \{L\}</math>)</code>               |
| power series to cont. fraction ( $L$ terms)   | <code>contfracinit(<math>S, \{L\}</math>)</code>                  |
| Padé approximant (deg. denom. $\leq B$ )      | <code>bestapprPade(<math>S, \{B\}</math>)</code>                  |

Elementary Arithmetic Functions

|                                          |                                                             |
|------------------------------------------|-------------------------------------------------------------|
| vector of binary digits of $ x $         | <code>binary(<math>x</math>)</code>                         |
| bit number $n$ of integer $x$            | <code>bittest(<math>x, n</math>)</code>                     |
| Hamming weight of integer $x$            | <code>hammingweight(<math>x</math>)</code>                  |
| digits of integer $x$ in base $B$        | <code>digits(<math>x, \{B = 10\}</math>)</code>             |
| sum of digits of integer $x$ in base $B$ | <code>sumdigits(<math>x, \{B = 10\}</math>)</code>          |
| integer from digits                      | <code>fromdigits(<math>v, \{B = 10\}</math>)</code>         |
| ceiling/floor/fractional part            | <code>ceil, floor, frac</code>                              |
| round $x$ to nearest integer             | <code>round(<math>x, \{\&amp;e\}</math>)</code>             |
| truncate $x$                             | <code>truncate(<math>x, \{\&amp;e\}</math>)</code>          |
| gcd/LCM of $x$ and $y$                   | <code>gcd(<math>x, y</math>), lcm(<math>x, y</math>)</code> |
| gcd of entries of a vector/matrix        | <code>content(<math>x</math>)</code>                        |

Primes and Factorization

|                                                            |                                                             |
|------------------------------------------------------------|-------------------------------------------------------------|
| extra prime table                                          | <code>addprimes()</code>                                    |
| add primes in $v$ to prime table                           | <code>addprimes(<math>v</math>)</code>                      |
| remove primes from prime table                             | <code>removeprimes(<math>v</math>)</code>                   |
| Chebyshev $\pi(x)$ , $n$ -th prime $p_n$                   | <code>primepi(<math>x</math>), prime(<math>n</math>)</code> |
| vector of first $n$ primes                                 | <code>primes(<math>n</math>)</code>                         |
| smallest prime $\geq x$                                    | <code>nextprime(<math>x</math>)</code>                      |
| largest prime $\leq x$                                     | <code>precprime(<math>x</math>)</code>                      |
| factorization of $x$                                       | <code>factor(<math>x, \{lim\}</math>)</code>                |
| ...selecting specific algorithms                           | <code>factorint(<math>x, \{flag = 0\}</math>)</code>        |
| $n = df^2$ , $d$ squarefree/fundamental                    | <code>core(<math>n, \{fl\}</math>), coredisc</code>         |
| certificate for (prime) $N$                                | <code>primecert(<math>N</math>)</code>                      |
| verifies a certificate $c$                                 | <code>primecertisvalid(<math>c</math>)</code>               |
| convert certificate to Magma/PRIMO                         | <code>primecertexport</code>                                |
| recover $x$ from its factorization                         | <code>factorback(<math>f, \{e\}</math>)</code>              |
| $x \in \mathbf{Z}$ , $ x  \leq X$ , $\gcd(N, P(x)) \geq N$ | <code>zncoppersmith(<math>P, N, X, \{B\}</math>)</code>     |
| divisors of $N$ in residue class $r$ mod $s$               | <code>divisorslensstra(<math>N, r, s</math>)</code>         |

Divisors and multiplicative functions

|                                                    |                                               |
|----------------------------------------------------|-----------------------------------------------|
| number of prime divisors $\omega(n)$ / $\Omega(n)$ | <code>omega(<math>n</math>), bigomega</code>  |
| divisors of $n$ / number of divisors $\tau(n)$     | <code>divisors(<math>n</math>), numdiv</code> |
| sum of ( $k$ -th powers of) divisors of $n$        | <code>sigma(<math>n, \{k\}</math>)</code>     |
| Möbius $\mu$ -function                             | <code>moebius(<math>x</math>)</code>          |
| Ramanujan's $\tau$ -function                       | <code>ramanujantau(<math>x</math>)</code>     |

Combinatorics

|                                                     |                                                           |
|-----------------------------------------------------|-----------------------------------------------------------|
| factorial of $x$                                    | <code>x!</code> or <code>factorial(<math>x</math>)</code> |
| binomial coefficient $\binom{x}{k}$                 | <code>binomial(<math>x, \{k\}</math>)</code>              |
| Bernoulli number $B_n$ as real/rational             | <code>bernreal(<math>n</math>), bernfrac</code>           |
| $[B_0, B_2, \dots B_{2k}]$                          | <code>bernvec(<math>k</math>)</code>                      |
| Bernoulli polynomial $B_n(x)$                       | <code>bernpol(<math>n, \{x\}</math>)</code>               |
| Euler numbers                                       | <code>eulerfrac, eulerreal, eulervec</code>               |
| Euler polynomial $E_n(x)$                           | <code>eulerpol(<math>n, \{x\}</math>)</code>              |
| Eulerian polynomial $A_n(x)$                        | <code>eulerianpol</code>                                  |
| Fibonacci number $F_n$                              | <code>fibonacci(<math>n</math>)</code>                    |
| Harmonic number $H_{n,r} = 1^{-r} + \dots + n^{-r}$ | <code>harmonic(<math>n, r</math>)</code>                  |
| Stirling numbers $s(n, k)$ and $S(n, k)$            | <code>stirling(<math>n, k, \{flag\}</math>)</code>        |

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(PARI-GP version 2.15.3)

|                                                                                                                                                                                                                                                                                                                        |                                                                    |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------|
| number of partitions of $n$                                                                                                                                                                                                                                                                                            | <code>numbpart(<math>n</math>)</code>                              |
| $k$ -th permutation on $n$ letters                                                                                                                                                                                                                                                                                     | <code>numtoperm(<math>n, k</math>)</code>                          |
| ...index $k$ of permutation $v$                                                                                                                                                                                                                                                                                        | <code>permtounum(<math>v</math>)</code>                            |
| order of permutation $p$                                                                                                                                                                                                                                                                                               | <code>permorder(<math>p</math>)</code>                             |
| signature of permutation $p$                                                                                                                                                                                                                                                                                           | <code>permsign(<math>p</math>)</code>                              |
| cyclic decomposition of permutation $p$                                                                                                                                                                                                                                                                                | <code>permcycles(<math>p</math>)</code>                            |
| <b>Multiplicative groups <math>(\mathbf{Z}/N\mathbf{Z})^*</math>, <math>\mathbf{F}_q^*</math></b>                                                                                                                                                                                                                      |                                                                    |
| Euler $\phi$ -function                                                                                                                                                                                                                                                                                                 | <code>eulerphi(<math>x</math>)</code>                              |
| multiplicative order of $x$ (divides $\phi$ )                                                                                                                                                                                                                                                                          | <code>znorder(<math>x, \{o\}</math>), fforder</code>               |
| primitive root mod $q$ / $x$ .mod                                                                                                                                                                                                                                                                                      | <code>znprimroot(<math>q</math>), fprimroot(<math>x</math>)</code> |
| structure of $(\mathbf{Z}/n\mathbf{Z})^*$                                                                                                                                                                                                                                                                              | <code>znstar(<math>n</math>)</code>                                |
| discrete logarithm of $x$ in base $g$                                                                                                                                                                                                                                                                                  | <code>znlog(<math>x, g, \{o\}</math>), fflag</code>                |
| Kronecker-Legendre symbol $(\frac{x}{y})$                                                                                                                                                                                                                                                                              | <code>kronecker(<math>x, y</math>)</code>                          |
| quadratic Hilbert symbol (at $p$ )                                                                                                                                                                                                                                                                                     | <code>hilbert(<math>x, y, \{p\}</math>)</code>                     |
| <b>Euclidean algorithm, continued fractions</b>                                                                                                                                                                                                                                                                        |                                                                    |
| CRT: solve $z \equiv x$ and $z \equiv y$                                                                                                                                                                                                                                                                               | <code>chinese(<math>x, y</math>)</code>                            |
| minimal $u, v$ so $xu + yv = \gcd(x, y)$                                                                                                                                                                                                                                                                               | <code>gcdext(<math>x, y</math>)</code>                             |
| half-gcd algorithm                                                                                                                                                                                                                                                                                                     | <code>halfgcd(<math>x, y</math>)</code>                            |
| continued fraction of $x$                                                                                                                                                                                                                                                                                              | <code>confrac(<math>x, \{b\}, \{lmax\}</math>)</code>              |
| last convergent of continued fraction $x$                                                                                                                                                                                                                                                                              | <code>confracpnqn(<math>x</math>)</code>                           |
| rational approximation to $x$ (den. $\leq B$ )                                                                                                                                                                                                                                                                         | <code>bestappr(<math>x, \{B\}</math>)</code>                       |
| recognize $x \in \mathbf{C}$ as polmod mod $T \in \mathbf{Z}[X]$                                                                                                                                                                                                                                                       | <code>bestapprnf(<math>x, T</math>)</code>                         |
| <b>Miscellaneous</b>                                                                                                                                                                                                                                                                                                   |                                                                    |
| integer square / $n$ -th root of $x$                                                                                                                                                                                                                                                                                   | <code>sqrtint(<math>x</math>), sqrtsint(<math>x, n</math>)</code>  |
| largest integer $e$ s.t. $b^e \leq b$ , $e = \lfloor \log_b(x) \rfloor$                                                                                                                                                                                                                                                | <code>logint(<math>x, b, \{\&amp;z\}</math>)</code>                |
| <b>Characters</b>                                                                                                                                                                                                                                                                                                      |                                                                    |
| Let $cyc = [d_1, \dots, d_k]$ represent an abelian group $G = \oplus (\mathbf{Z}/d_j\mathbf{Z}) \cdot g_j$ or any structure $G$ affording a .cyc method; e.g. <code>znstar(<math>q, 1</math>)</code> for Dirichlet characters. A character $\chi$ is coded by $[c_1, \dots, c_k]$ such that $\chi(g_j) = e(n_j/d_j)$ . |                                                                    |
| $\chi \cdot \psi$ ; $\chi^{-1}$ ; $\chi \cdot \psi^{-1}$ ; $\chi^k$                                                                                                                                                                                                                                                    | <code>charmul, charconj, chardiv, charpow</code>                   |
| order of $\chi$                                                                                                                                                                                                                                                                                                        | <code>charorder(<math>cyc, \chi</math>)</code>                     |
| kernel of $\chi$                                                                                                                                                                                                                                                                                                       | <code>charker(<math>cyc, \chi</math>)</code>                       |
| $\chi(x)$ , $G$ a GP group structure                                                                                                                                                                                                                                                                                   | <code>chareval(<math>G, \chi, x, \{z\}</math>)</code>              |
| Galois orbits of characters                                                                                                                                                                                                                                                                                            | <code>chargalois(<math>G</math>)</code>                            |
| <b>Dirichlet Characters</b>                                                                                                                                                                                                                                                                                            |                                                                    |
| initialize $G = (\mathbf{Z}/q\mathbf{Z})^*$                                                                                                                                                                                                                                                                            | <code>G = znstar(<math>q, 1</math>)</code>                         |
| convert datum $D$ to $[G, \chi]$                                                                                                                                                                                                                                                                                       | <code>znchar(<math>D</math>)</code>                                |
| is $\chi$ odd?                                                                                                                                                                                                                                                                                                         | <code>zncharisodd(<math>G, \chi</math>)</code>                     |
| real $\chi \rightarrow$ Kronecker symbol $(D/\cdot)$                                                                                                                                                                                                                                                                   | <code>znchartokronecker(<math>G, \chi</math>)</code>               |
| conductor of $\chi$                                                                                                                                                                                                                                                                                                    | <code>zncharconductor(<math>G, \chi</math>)</code>                 |
| $[G_0, \chi_0]$ primitive attached to $\chi$                                                                                                                                                                                                                                                                           | <code>znchartoprimitive(<math>G, \chi</math>)</code>               |
| induce $\chi \in \hat{G}$ to $\mathbf{Z}/N\mathbf{Z}$                                                                                                                                                                                                                                                                  | <code>zncharinduce(<math>G, \chi, N</math>)</code>                 |
| $\chi p$                                                                                                                                                                                                                                                                                                               | <code>znchardecompose(<math>G, \chi, p</math>)</code>              |
| $\prod_p  (Q, N) \chi p$                                                                                                                                                                                                                                                                                               | <code>znchardecompose(<math>G, \chi, Q</math>)</code>              |
| complex Gauss sum $G_a(\chi)$                                                                                                                                                                                                                                                                                          | <code>znchargauss(<math>G, \chi</math>)</code>                     |
| <b>Conrey labelling</b>                                                                                                                                                                                                                                                                                                |                                                                    |
| Conrey label $m \in (\mathbf{Z}/q\mathbf{Z})^* \rightarrow$ character                                                                                                                                                                                                                                                  | <code>znconreychar(<math>G, m</math>)</code>                       |
| character $\rightarrow$ Conrey label                                                                                                                                                                                                                                                                                   | <code>znconreyexp(<math>G, \chi</math>)</code>                     |
| log on Conrey generators                                                                                                                                                                                                                                                                                               | <code>znconreylog(<math>G, m</math>)</code>                        |
| conductor of $\chi$ ( $\chi_0$ primitive)                                                                                                                                                                                                                                                                              | <code>znconreyconductor(<math>G, \chi, \{\chi_0\}</math>)</code>   |

True-False Tests

|                                                  |                                                          |
|--------------------------------------------------|----------------------------------------------------------|
| is $x$ the disc. of a quadratic field?           | <code>isfundamental(<math>x</math>)</code>               |
| is $x$ a prime?                                  | <code>isprime(<math>x</math>)</code>                     |
| is $x$ a strong pseudo-prime?                    | <code>ispseudoprime(<math>x</math>)</code>               |
| is $x$ square-free?                              | <code>issquarefree(<math>x</math>)</code>                |
| is $x$ a square?                                 | <code>issquare(<math>x, \{\&amp;n\}</math>)</code>       |
| is $x$ a perfect power?                          | <code>ispower(<math>x, \{k\}, \{\&amp;n\}</math>)</code> |
| is $x$ a perfect power of a prime? ( $x = p^n$ ) | <code>isprimepower(<math>x, \&amp;n</math>)</code>       |
| ... of a pseudoprime?                            | <code>ispseudoprimepower(<math>x, \&amp;n</math>)</code> |
| is $x$ powerful?                                 | <code>ispowerful(<math>x</math>)</code>                  |
| is $x$ a totient? ( $x = \varphi(n)$ )           | <code>istotient(<math>x, \{\&amp;n\}</math>)</code>      |
| is $x$ a polygonal number? ( $x = P(s, n)$ )     | <code>ispolygonal(<math>x, s, \{\&amp;n\}</math>)</code> |
| is $pol$ irreducible?                            | <code>polisirreducible(<math>pol</math>)</code>          |

Graphic Functions

|                                              |                                                                         |
|----------------------------------------------|-------------------------------------------------------------------------|
| crude graph of $expr$ between $a$ and $b$    | <code>plot(<math>X = a, b, expr</math>)</code>                          |
| <b>High-resolution plot</b> (immediate plot) |                                                                         |
| plot $expr$ between $a$ and $b$              | <code>plotoh(<math>X = a, b, expr, \{flag\}, \{n\}</math>)</code>       |
| plot points given by lists $lx, ly$          | <code>plotthraw(<math>lx, ly, \{flag\}</math>)</code>                   |
| terminal dimensions                          | <code>plotsizes()</code>                                                |
| <b>Rectwindow functions</b>                  |                                                                         |
| init window $w$ , with size $x, y$           | <code>plotinit(<math>w, x, y</math>)</code>                             |
| erase window $w$                             | <code>plotkill(<math>w</math>)</code>                                   |
| copy $w$ to $w_2$ with offset $(dx, dy)$     | <code>plotcopy(<math>w, w_2, dx, dy</math>)</code>                      |
| slice contents of $w$                        | <code>plotclip(<math>w</math>)</code>                                   |
| scale coordinates in $w$                     | <code>plotscale(<math>w, x_1, x_2, y_1, y_2</math>)</code>              |
| plotoh in $w$                                | <code>plotrecth(<math>w, X = a, b, expr, \{flag\}, \{n\}</math>)</code> |
| plotthraw in $w$                             | <code>plotrectthraw(<math>w, data, \{flag\}</math>)</code>              |
| draw window $w_1$ at $(x_1, y_1), \dots$     | <code>plotdraw(<math>[[w_1, x_1, y_1], \dots]</math>)</code>            |

Low-level Rectwindow Functions

|                                         |                                                          |
|-----------------------------------------|----------------------------------------------------------|
| set current drawing color in $w$ to $c$ | <code>plotcolor(<math>w, c</math>)</code>                |
| current position of cursor in $w$       | <code>plotcursor(<math>w</math>)</code>                  |
| write $s$ at cursor's position          | <code>plotstring(<math>w, s</math>)</code>               |
| move cursor to $(x, y)$                 | <code>plotmove(<math>w, x, y</math>)</code>              |
| move cursor to $(x + dx, y + dy)$       | <code>plotrmove(<math>w, dx, dy</math>)</code>           |
| draw a box to $(x_2, y_2)$              | <code>plotbox(<math>w, x_2, y_2</math>)</code>           |
| draw a box to $(x + dx, y + dy)$        | <code>plotrbox(<math>w, dx, dy</math>)</code>            |
| draw polygon                            | <code>plotlines(<math>w, lx, ly, \{flag\}</math>)</code> |
| draw points                             | <code>plotpoints(<math>w, lx, ly</math>)</code>          |
| draw line to $(x + dx, y + dy)$         | <code>plotrline(<math>w, dx, dy</math>)</code>           |
| draw point $(x + dx, y + dy)$           | <code>plotrpoint(<math>w, dx, dy</math>)</code>          |

Convert to Postscript or Scalable Vector Graphics

|                                         |                                                                          |
|-----------------------------------------|--------------------------------------------------------------------------|
| The format $f$ is either "ps" or "svg". |                                                                          |
| as plotoh                               | <code>plotexport(<math>f, X = a, b, expr, \{flag\}, \{n\}</math>)</code> |
| as plotthraw                            | <code>plotthrawexport(<math>f, lx, ly, \{flag\}</math>)</code>           |
| as plotdraw                             | <code>plotexport(<math>f, [[w_1, x_1, y_1], \dots]</math>)</code>        |